VNA Tools II
S-parameter uncertainty calculation

Michael Wollensack, Johannes Hoffmann,
Juerg Ruefenacht, Markus Zeier

METAS Swiss Federal Office of Metrology

22. June 2012

Outline

Introduction

Problem

Computation of the uncertainties of S-parameter measurements.

Solution

Set up a measurement model for the Vector Network Analyzer and propagate all uncertainties through this model.

VNA Measurement Model

The following equation describes the in VNA Tools II used N-port VNA measurement model. All bold variables are S-parameter matrices and i is the measurement index.

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)}\right) \oplus \left[\left(\mathbf{E} + \mathbf{D}^{(i)}\right) \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)}\right]\right]\right]$$

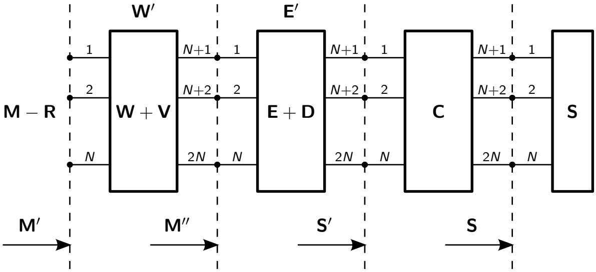


Figure: VNA Measurement Model

VNA Measurement Model - Raw Data

M denotes the raw data measured by the VNA.
It changes from measurement to measurement.

R denotes the noise and linearity influences.
It changes from measurement to measurement.

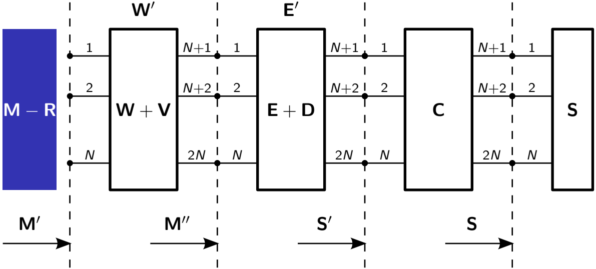


Figure: VNA Measurement Model

VNA Measurement Model - Switch Terms

W denotes the switch terms.
It's constant during an entire calibration.

V denotes the drift of the switch terms.
It changes from measurement to measurement.

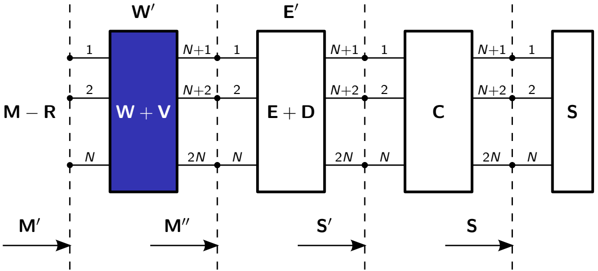


Figure: VNA Measurement Model

VNA Measurement Model - Calibration Error Terms

E denotes the calibration error terms.
It's constant during an entire calibration.

D denotes the drift of the calibration error terms.
It changes from measurement to measurement.

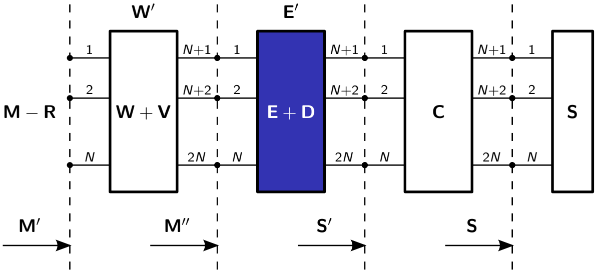


Figure: VNA Measurement Model

VNA Measurement Model - Cable and Connector

C denotes the cable stability and connector repeatability influences. It changes for every new connection or cable movement.

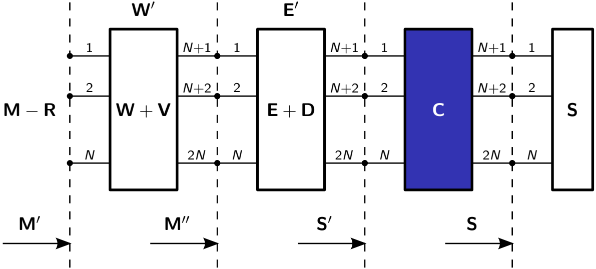


Figure: VNA Measurement Model

VNA Measurement Model - Error Corrected Data

S denotes the error corrected data or the calibration kit standard definitions. It changes if a new device is connected.

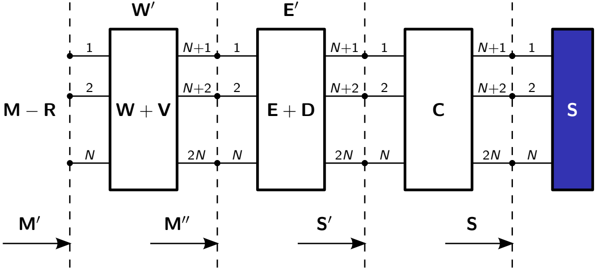


Figure: VNA Measurement Model

Measurement Uncertainty

The following non correctable influences affect the S-parameter measurements:

1. Noise floor and trace noise of the VNA
2. Linearity of the VNA
3. Drift of switch and error terms of the calibration
4. Cable stability
5. Connector repeatability
6. Definition of the calibration standards

All of these influences are frequency dependent and are either of random or systematic nature.

Uncertainty Influences - Noise and Linearity

Noise

- Uncorrelated for each measurement.
- Depends on the definition of the noise floor and the trace noise of the VNA.

Linearity

- Correlated for each measurement.
- Depends on the linearity definition of the VNA.

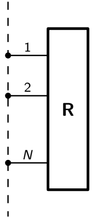


Figure: Noise and linearity in the VNA measurement model

Uncertainty Influences - Drift of Switch and Error Terms

Drift

- Considered as uncorrelated for each measurement.
- Depends on the drift definitions of the switch terms and error terms (directivity, tracking, match and isolation) of the VNA.

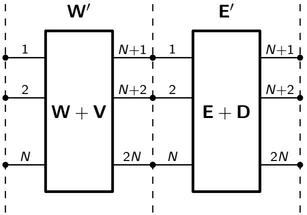


Figure: Drift of switch and error terms in the VNA measurement model

Uncertainty Influences - Cable and Connector

Cable

- Uncorrelated for each new cable position.
- Depends on the cable stability definition.

Connector

- Uncorrelated for each new connection.
- Depends on the connector repeatability definition.

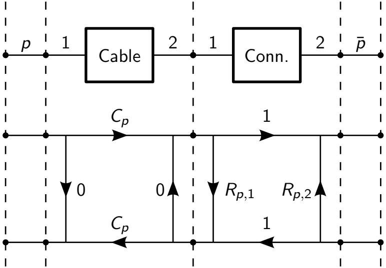


Figure: Cable stability and connector repeatability 2-ports for one port in the VNA measurement model

Uncertainty Influences - Calibration Standard

Agilent Model Standard

- Open and Short have specified phase deviation in deg. Magnitude deviation assumed to be the same as the phase deviation.
- Load has specified return loss in dB.

Databased Standard

- Uncertainties explicitly stated for each data point.

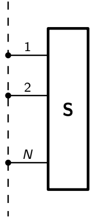


Figure: Calibration standard definition in the VNA measurement model

Uncertainty Propagation

- The uncertainty influences are defined by **R**, **V**, **D**, **C** and **S**.
- VNA measurement model:

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)} \right) \oplus \left[\left(\mathbf{E} + \mathbf{D}^{(i)} \right) \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)} \right] \right] \right]$$

- Calibration and error correction are based on the above equation.
- The uncertainty influences are linearly propagated through calibration and error correction using METAS UncLib.

METAS UncLib

UncLib is a generic measurement uncertainty calculator.

The user specifies

- input quantities $\mathbf{X}$ with input covariance matrix $\mathbf{V}_X$
- measurement model $\mathbf{f}$

METAS UncLib computes

- output quantities $\mathbf{Y} = \mathbf{f}(\mathbf{X})$
- Jacobi matrix $\mathbf{J}_{YX}$ of $\mathbf{f}$ using automatic differentiation
- output covariance matrix $\mathbf{V}_Y = \mathbf{J}_{YX}\mathbf{V}_X\mathbf{J}_{YX}'$

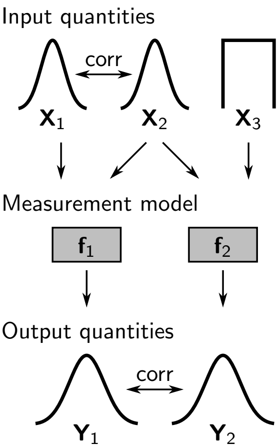


Figure: METAS UncLib

Visualization - Graph

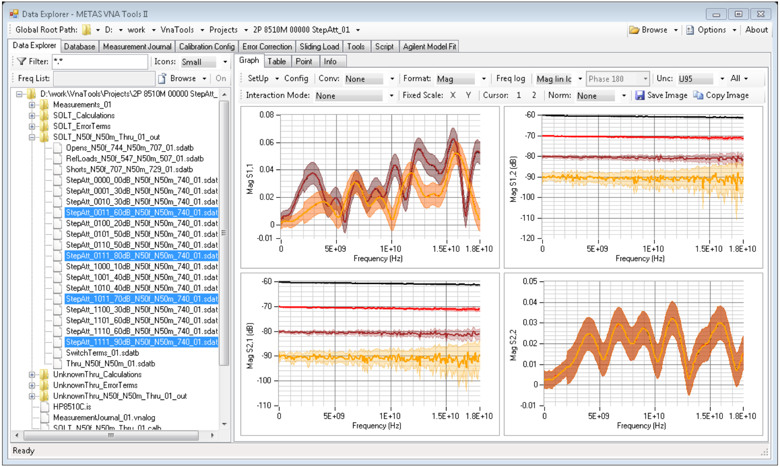
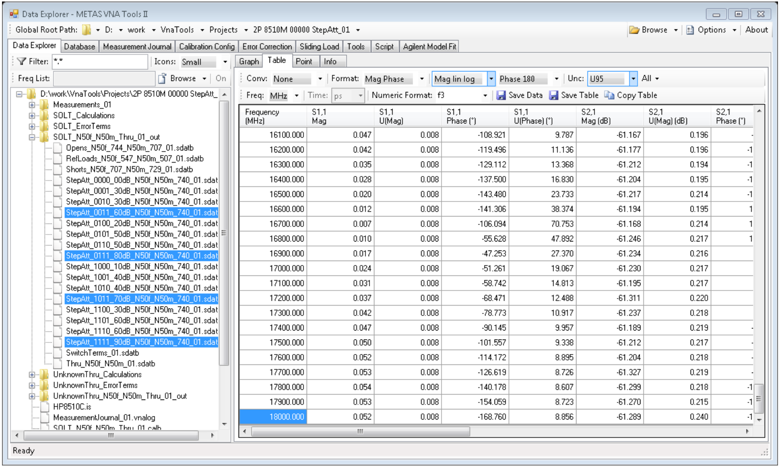


Figure: Graph shows a graphical visualization of multiple files.

Visualization - Table



Frequency (MHz)	S1,1 Mag	S1,1 U(Mag)	S1,1 Phase (°)	S1,1 U(Phase) (°)	S2,1 Mag (dB)	S2,1 U(Mag) (dB)	S2,1 Phase (°)
16100.000	0.047	0.008	-108.921	9.787	-61.167	0.196	-
16200.000	0.042	0.008	-119.496	11.136	-61.177	0.196	-1
16300.000	0.035	0.008	-129.112	13.368	-61.212	0.194	-1
16400.000	0.028	0.008	-137.500	16.830	-61.204	0.195	-1
16500.000	0.020	0.008	-143.480	23.733	-61.217	0.214	-1
16600.000	0.012	0.008	-141.306	38.374	-61.194	0.195	1
16700.000	0.007	0.008	-106.094	70.753	-61.168	0.214	1
16800.000	0.010	0.008	-55.628	47.892	-61.246	0.217	1
16900.000	0.017	0.008	-47.253	27.370	-61.234	0.216	
17000.000	0.024	0.008	-51.261	19.067	-61.230	0.217	
17100.000	0.031	0.008	-58.742	14.813	-61.195	0.217	
17200.000	0.037	0.008	-68.471	12.488	-61.311	0.220	
17300.000	0.042	0.008	-78.773	10.917	-61.237	0.218	
17400.000	0.047	0.008	-90.145	9.957	-61.189	0.219	-
17500.000	0.050	0.008	-101.557	9.338	-61.212	0.217	-
17600.000	0.052	0.008	-114.172	8.895	-61.204	0.218	-
17700.000	0.053	0.008	-126.619	8.726	-61.327	0.219	-
17800.000	0.054	0.008	-140.178	8.607	-61.299	0.218	-1
17900.000	0.053	0.008	-154.059	8.723	-61.270	0.215	-1
18000.000	0.052	0.008	-168.760	8.856	-61.289	0.240	-1

Figure: Table shows a tabular visualization of a single file.

Visualization - Point

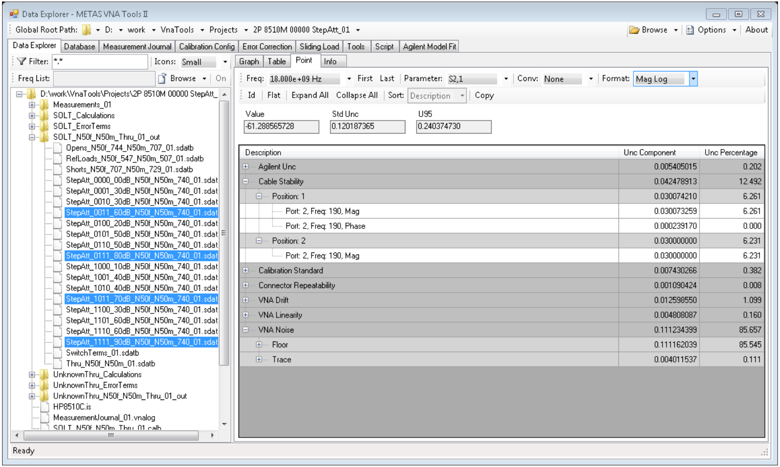


Figure: Point shows an uncertainty budget for one frequency point and one parameter of a single file.

Example - Comparison

Comparison of two different calibration techniques starting from the same raw data.

Calibration standards

- Short, open, load at port 1
- Thru connection between port 1 and 2

Calibration techniques

- Quick Short Open Load Thru (QSOLT)
- Optimization

Device under test

- 20 dB attenuator

Example - QSOLT Results

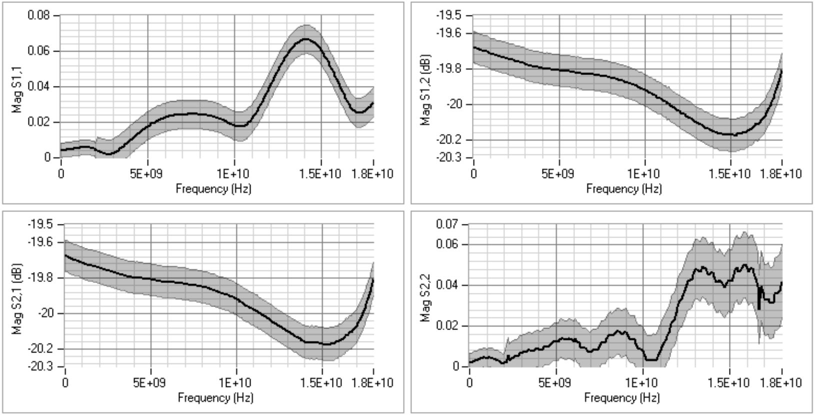


Figure: S-parameters of 20 dB attenuator using QSOLT calibration.

Example - Optimization Results

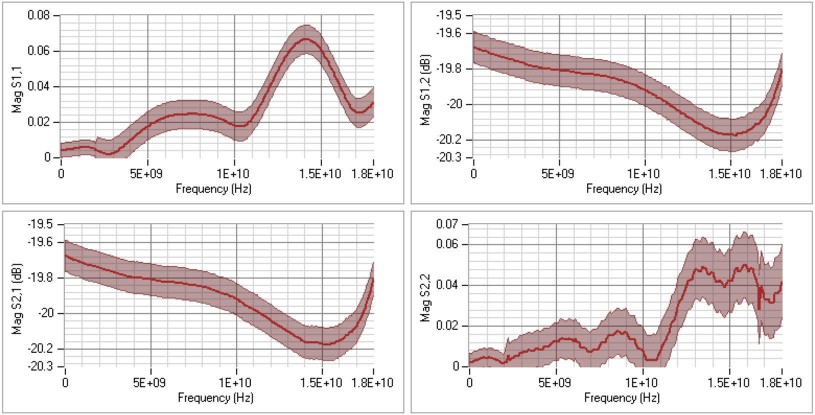


Figure: S-parameters of 20 dB attenuator using optimization calibration. The observed differences between the resulting uncertainties are lower than 0.3 % over the whole frequency range and all four S-parameters.

Conclusion

- Measurement model for a N-port VNA.
- Definition of influences that affect the measurement.
- Linear propagation of uncertainties through the measurement model.
- Visualization of S-parameter data with uncertainty budget.

http://www.metas.ch/vnatools

Optimization

The following equation describes the in VNA Tools II used N-port VNA measurement model.

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)} \right) \oplus \left[\left(\mathbf{E} + \mathbf{D}^{(i)} \right) \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)} \right] \right] \right]$$

The inverse function of the above equation can be used for error correction.

$$\mathbf{S}^{(i)} = \left[\left[\left(\mathbf{M}^{(i)} - \mathbf{R}^{(i)} \right) \ominus \left(\mathbf{W} + \mathbf{V}^{(i)} \right) \right] \ominus \left(\mathbf{E} + \mathbf{D}^{(i)} \right) \right] \ominus \mathbf{C}^{(i)}$$

The optimizer minimizes the following objective function for all measurements.

$$\left[\left[\left[\left(\mathbf{M}^{(i)} - \mathbf{R}^{(i)} \right) \ominus \left(\mathbf{W} + \mathbf{V}^{(i)} \right) \right] \ominus \left(\mathbf{E} + \mathbf{D}^{(i)} \right) \right] \ominus \mathbf{C}^{(i)} \right] - \mathbf{S}^{(i)}$$

Optimization - Weighting

The following equation describes the objective function f where $\mathbf{X}$ are the variable optimization parameters and $\mathbf{P}$ are the constant optimization parameters.

Objective function	$\mathbf{F} = f(\mathbf{X}, \mathbf{P})$
Covariance	$\mathbf{C}_F = \mathbf{J}_{F,P}\mathbf{C}_P\mathbf{J}'_{F,P}$
Optimization problem	$\min\limits_{\mathbf{X}\in\mathbb{R}} \left(\mathbf{F}\mathbf{C}_F^{-1}\mathbf{F}'\right)$
or	$\min\limits_{\mathbf{X}\in\mathbb{R}} \left(\mathbf{G}\mathbf{G}'\right)$ with $\mathbf{G} = \mathbf{F}\mathbf{W}_F$

The weights $\mathbf{W}_F$ are computed with the eigenvalue decomposition of the covariance $\mathbf{C}_F$.

Optimization - Uncertainty Propagation

The Jacobi matrix **X** to **P** at the point of the solution is described with the following equation.

$$\mathbf{J}_{X,P} = \underbrace{\left(\mathbf{J}'_{G,X}\mathbf{J}_{G,X}\right)^{-1}\mathbf{J}'_{G,X}}_{\text{pseudoinverse of } \mathbf{J}_{G,X}}\mathbf{J}_{G,P}$$

It can be used for the uncertainty propagation.

Optimization - Results

- The in VNA Tools II implemented optimization calibration can be used for over-determined calibration.
- It can be used for any calibration type, like SOLT, QSLOT, Unknown Thru, LRL ...

Validation

- METAS VNA Tools II - Math Reference can be used to verify the program.
- METAS UncLib used for uncertainty propagation.
  - Only derivatives of elementary functions are programmed. Elementary functions are easy to check.
  - Overloaded operators hide the complexity from the user. If the value is right, the uncertainty is right.
- SOLT, QSOLT, UThru and others compared to optimization calibration. Two mathematically independent ways yield same results and uncertainties.
- User has the possibility to check if the uncertainty budget is plausible.
- VNA measurement model is an assumption. Other methods have to assume as well a model.