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VNA Tools II: S-parameter uncertainty calculation

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Abstract—This paper describes a software, METAS VNA Tools II, which is designed to compute uncertainties of coaxial S-parameter measurements. A bottom-up concept is used. Thus basic influence quantities are propagated through the calibration of the vector network analyzer to the S-parameters of a device under test. METAS UncLib is used for the linear propagation of uncertainties. The result is not only an uncertainty region but a list of uncertainty contributions with correlations. Thus the uncertainties can be propagated into eventual post-processing steps. In the present paper the concept has been verified by computing uncertainties of a calibration with a traditional quick short open load thru algorithm and an algorithm which involves optimization. The observed differences between the resulting uncertainties are lower than 0.3 percent, which can be explained by numerical inaccuracies.

Index Terms—Vector Network Analyzer, S-parameters, Calibration, Uncertainty, Traceability

I. INTRODUCTION

The problem treated here is the computation of the uncertainties of coaxial S-parameter measurements, see [1]. Up to now a common way to handle uncertainties of vector network analyzer (VNA) measurements is the ripple technique described in [2]. This approach uses precision airlines and other physical standards to extract the residual errors of a calibrated VNA system. It is based on the assumption that the precision transmission line has zero reflection. However, the systematic reflection coefficients of connectors make this assumption invalid, see [3].

More sophisticated error models have been implemented in other techniques, e.g. StatistiCAL [4].

A fundamental solution is to use a bottom-up concept as it has been implemented in METAS VNA Tools II. One can set up a measurement model for the VNA and propagate all uncertainties through this model. This methodology is described in the Guide to the Expression of Uncertainty in Measurement (GUM) [5], [6].

II. VNA MEASUREMENT MODEL

A very well known measurement model for VNAs is described in [7]. However the model used here is slightly different. Our model is an N -port model and uncertainties are assigned to the error terms instead of mapping them to residual terms. Furthermore the error terms are allowed to drift.

Figure 1 shows the measurement model of a N -port VNA as used in VNA Tools II.

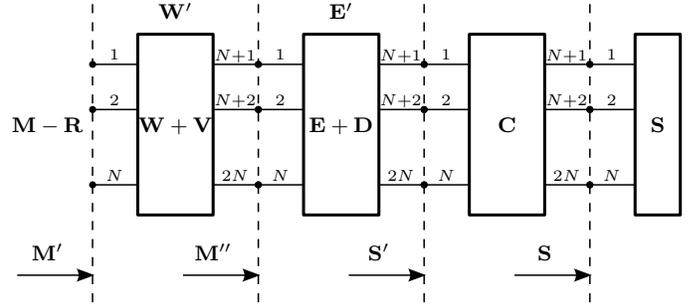


Fig. 1. VNA Measurement Model

The following equation describes the measurement model.

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)} \right) \oplus \left[\left(\mathbf{E} + \mathbf{D}^{(i)} \right) \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)} \right] \right] \right] \quad (1)$$

All bold variables are S-parameter matrices. \mathbf{M} , \mathbf{R} and \mathbf{S} are N -ports, the other bold variables are $2N$ -ports and i is the measurement index.

- \mathbf{M} denotes the raw data measured by the VNA.
- \mathbf{R} denotes the noise and linearity influences.
- \mathbf{W} denotes the switch terms.
- \mathbf{V} denotes the drift of the switch terms.
- \mathbf{E} denotes the calibration error terms.
- \mathbf{D} denotes the drift of the calibration error terms.
- \mathbf{C} denotes the cable stability and connector repeatability influences.
- \mathbf{S} denotes the error corrected data or the calibration kit standard definitions.

\mathbf{M} , \mathbf{R} , \mathbf{V} and \mathbf{D} change from measurement to measurement. \mathbf{W} and \mathbf{E} are constant during an entire calibration. \mathbf{C} changes for every new connection or cable movement. \mathbf{S} changes if a new device is connected.

The operator \oplus denotes the cascading of two S-parameter sets, as described in [8].

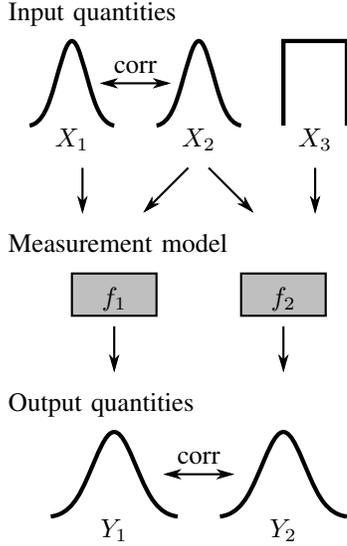


Fig. 2. Illustration of multivariate uncertainty propagation: The uncertainties and correlations of basic input quantities X_1 , X_2 , X_3 are propagated through a measurement model resulting in uncertainties and correlations of multiple output quantities Y_1 , Y_2 , see [6].

uncertainty is done in the background in an automated way. METAS UncLib keeps automatically track of the derivatives with respect to the input quantities. Formally this means that METAS UncLib computes the output quantities $\mathbf{Y} = \mathbf{f}(\mathbf{X})$ and the Jacobi matrix \mathbf{J}_{YX} of \mathbf{f} that contains the derivatives of the components of \mathbf{Y} with respect to the components of \mathbf{X} . On demand METAS UncLib can compute the output covariance matrix $\mathbf{V}_Y = \mathbf{J}_{YX} \mathbf{V}_X \mathbf{J}_{YX}'$. Further information about the technique behind METAS UncLib can be found in [15].

C. Optimization

The following equation describes the objective function f where \mathbf{X} are the variable optimization parameters and \mathbf{P} are the constant optimization parameters.

$$\mathbf{F} = f(\mathbf{X}, \mathbf{P}) \quad (6)$$

For the weighting of the optimization problem the covariance of the objective function is needed

$$\mathbf{C}_F = \mathbf{J}_{F,P} \mathbf{C}_P \mathbf{J}_{F,P}' \quad (7)$$

where \mathbf{C}_P is the covariance of the constant optimization parameters and $\mathbf{J}_{F,P}$ is the Jacobi matrix \mathbf{F} to \mathbf{P} . The optimization problem is described with the following expression.

$$\min_{\mathbf{X} \in \mathbb{R}} (\mathbf{F} \mathbf{C}_F^{-1} \mathbf{F}') \quad (8)$$

One can introduce $\mathbf{G} = \mathbf{F} \mathbf{W}_F$, then the optimization problem becomes

$$\min_{\mathbf{X} \in \mathbb{R}} (\mathbf{G} \mathbf{G}') \quad (9)$$

where the weights \mathbf{W}_F are computed from the covariance of the objective function \mathbf{C}_F using the eigenvalue decomposition.

For the uncertainty propagation at the point of the solution the Jacobi matrix \mathbf{X} to \mathbf{P} is described with the following equation.

$$\mathbf{J}_{X,P} = (\mathbf{J}'_{G,X} \mathbf{J}_{G,X})^{-1} \mathbf{J}'_{G,X} \mathbf{J}_{G,P} \quad (10)$$

IV. VISUALIZATION

VNA Tools II supports different view modes to visualize measurement data:

Graph shows a graphical visualization of multiple files, see Figure 3.

Table shows a tabular visualization of a single file.

Point shows an uncertainty budget for one frequency point and one parameter of a single file.

There are three different uncertainty modes:

None hides the uncertainty.

Std shows the standard uncertainty (67% coverage factor, $k = 1$).

U95 shows the expanded uncertainty (95% coverage factor, $k = 2$).

V. EXAMPLE

In this example the concept of VNA Tools II will be verified by computing uncertainties of a 20 dB attenuator using two different calibration techniques starting from the same raw data. The following calibration standards have been measured:

- A short, an open and a load at port 1 are measured for the reflection calibration of port 1.
- A thru connection between port 1 and 2 is used to copy the error box from port 1 to port 2.

In addition the switch terms are measured and the raw measurements are corrected for the imperfect switch.

The first calibration is a traditional quick short open load thru (QSOLT) algorithm. The second calibration is an algorithm which involves optimization. In this example we use the 7 error-term model but in general VNA Tools II can handle arbitrary error models. The verification for this example is based on the following reasoning: If the number of measurements is the same as the number of unknowns, then the optimization calibration should give about the same results and uncertainties as the QSOLT calibration. Small numerical differences can be expected due to the different ways the solutions are derived, the analytical solution of the QSOLT algorithm vs the numerical solution of the optimization code.

Table I shows the uncertainty budget of the reflection at port 1 of the 20 dB attenuator and Table II shows the uncertainty budget of the transmission from port 1 to port 2. These uncertainty budgets are from the QSOLT calibration. Those from the optimization calibration look very similar.

The observed differences between the resulting uncertainties of the 20 dB attenuator are lower than 0.3 percent over the whole frequency range and all four S-parameters. This can be considered as an excellent agreement.

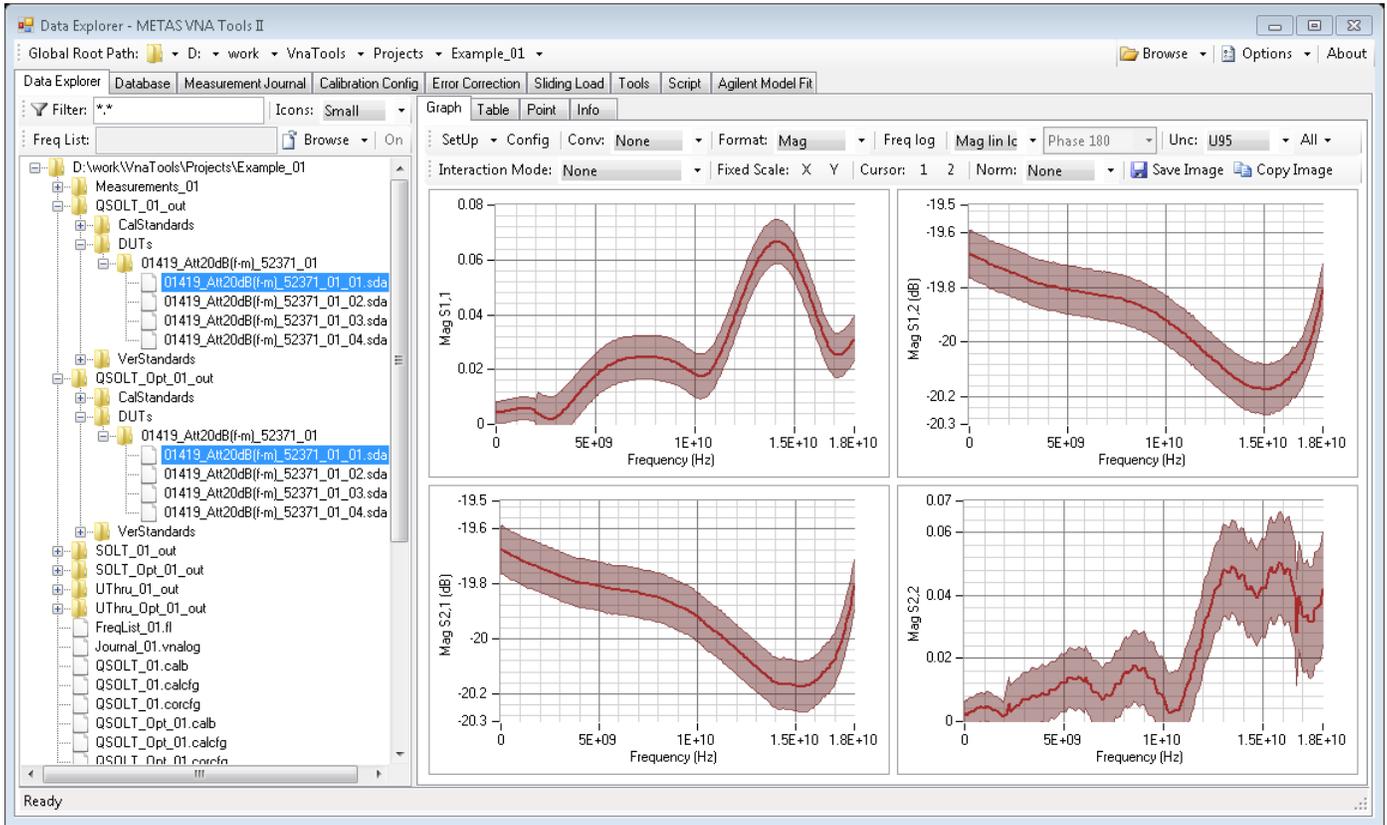


Fig. 3. VNA Tools II / Data Explorer / Graph

TABLE I

UNCERTAINTY BUDGET, FREQUENCY: 18 GHz, PARAMETER: S1,1 MAGNITUDE, VALUE: 0.031, U95: 0.008

| Description | Unc Component | Unc Percentage |
|--|---------------|----------------|
| Calibration Standard / Load / Type N (50) female load / Real | 0.003568 | 76.430 |
| Calibration Standard / Load / Type N (50) female load / Imag | 0.001805 | 19.553 |
| Connector Repeatability / Connection: 1 (Load) / Port: 1, Freq: 18 GHz / Real | 0.000447 | 1.201 |
| Connector Repeatability / Connection: 9 (DUT) / Port: 1, Freq: 18 GHz / Real | 0.000444 | 1.185 |
| Connector Repeatability / Connection: 1 (Load) / Port: 1, Freq: 18 GHz / Imag | 0.000231 | 0.320 |
| Connector Repeatability / Connection: 9 (DUT) / Port: 1, Freq: 18 GHz / Imag | 0.000229 | 0.316 |
| Calibration Standard / Open / Type N (50) female open / Mag | 0.000210 | 0.265 |
| VNA Drift / Directivity / Measurement: 4 (Load) / Port: 1, Freq: 18 GHz / Imag | 0.000174 | 0.183 |
| VNA Drift / Directivity / Measurement: 23 (DUT) / Port: 1, Freq: 18 GHz / Imag | 0.000171 | 0.176 |
| VNA Drift / Directivity / Measurement: 4 (Load) / Port: 1, Freq: 18 GHz / Real | 0.000143 | 0.123 |
| VNA Drift / Directivity / Measurement: 23 (DUT) / Port: 1, Freq: 18 GHz / Real | 0.000143 | 0.122 |
| Calibration Standard / Short / Type N (50) female short / Mag | 0.000132 | 0.105 |

TABLE II

UNCERTAINTY BUDGET, FREQUENCY: 18 GHz, PARAMETER: S2,1 MAGNITUDE, VALUE: -19.805 dB, U95: 0.093 dB

| Description | Unc Component | Unc Percentage |
|---|---------------|----------------|
| Cable Stability / Position: 1 (Thru) / Port: 2, Freq: 18 GHz / Mag | 0.030000 dB | 41.866 |
| Cable Stability / Position: 4 (DUT) / Port: 2, Freq: 18 GHz / Mag | 0.030000 dB | 41.866 |
| VNA Drift / Tracking / Measurement: 23 (DUT) / Port: 2, Freq: 18 GHz / Mag | 0.011385 dB | 6.029 |
| VNA Drift / Tracking / Measurement: 19 (Thru) / Port: 2, Freq: 18 GHz / Mag | 0.011385 dB | 6.029 |
| VNA Noise / Trace / Measurement: 23 (DUT) / S2,1, Freq: 190 / Mag | 0.006364 dB | 1.884 |
| VNA Noise / Trace / Measurement: 19 (Thru) / S2,1, Freq: 190 / Mag | 0.006284 dB | 1.837 |
| Calibration Standard / Open / Type N (50) female open / Mag | 0.001662 dB | 0.129 |

VI. CONCLUSION

VNA Tools II is a metrology software for VNA that supports the treatment of measurement uncertainty in accordance with internationally accepted guidance. It is based on a new VNA measurement model for a N -port Vector Network Analyzer. A comprehensive set of influences that affect the measurements can be defined and are recorded during the measurement process. All uncertainties are linearly propagated through the measurement model taking correlations into account. VNA Tools II can visualize S-parameter data with uncertainties and provides a complete uncertainty budget for each parameter at each frequency point.

SOFTWARE

The VNA Tools II software package implementing the VNA measurement model described here can be downloaded online: <http://www.metas.ch/vnatools>.

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REFERENCES

- [1] K. Wong, "Traceability of vector network analyzer measurements", in *72th ARFTG Conf. Dig.*, pp. 157-167, December 2008
- [2] *Guidelines on the Evaluation of Vector Network Analysers (VNA)*, European Association of National Metrology Institutes (EURAMET), cg-12, Version 2.0, March 2011.
- [3] J. Hoffmann, P. Leuchtman, J. Ruefenacht, and K. Wong, "S-parameters of Slotted and Slotless Coaxial Connectors", in *74th ARFTG Conf. Dig.*, pp. 1-5, December 2009
- [4] D. F. Williams, J. C. M. Wang, and U. Arz, "An optimal vector-network-analyzer calibration algorithm", *IEEE Trans. Microwave Theory & Tech.*, vol. 51, no. 12, pp. 987-993, December 2003.
- [5] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, *Evaluation of measurement data - Guide to the expression of uncertainty in measurement*, International Organization for Standardization (ISO), Online: <http://www.bipm.org/en/publications/guides/gum.html>, September 2008
- [6] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, *Evaluation of measurement data - Supplement 2 to the "Guide to the expression of uncertainty in measurement" - Extension to any number of output quantities*, International Organization for Standardization (ISO), Online: <http://www.bipm.org/en/publications/guides/gum.html>, October 2011
- [7] D. K. Rytting, "Network Analyzer Accuracy Overview", in *58th ARFTG Conf. Dig.*, pp. 1-13, November 2001
- [8] A. Ferrero, U. Pisani, and K. J. Kerwin, "A new implementation of a multiport automatic network analyzer", *IEEE Trans. Microwave Theory & Tech.*, vol. 40, no. 11, pp. 2078-2085, November 1992.
- [9] A. Ferrero, and U. Pisani, "QSOLT: a new fast calibration algorithm for two port S-parameter measurements", in *38th ARFTG Conf. Dig.*, pp. 15-24, December 1991.
- [10] A. Ferrero, and U. Pisani, "Two-port network analyzer calibration using an unknown thru", *IEEE Microwave & Guided Wave Letters*, vol. 2, no. 12, pp. 505-507, December 1992.
- [11] G. F. Engen, and C. A. Hoer, "Thru-reflect-line: An improved technique for calibrating the dual six-port automatic network analyzer", *IEEE Trans. Microwave Theory & Tech.*, vol. 27, no. 12, pp. 987-993, December 1979.
- [12] R. B. Marks, "Formulations of the basic vector network analyzer error model including switch-terms", in *50th ARFTG Conf. Dig.*, pp. 115-126, December 1997
- [13] J. Hoffmann, P. Leuchtman, J. Ruefenacht, and R. Vahldieck, "A stable bayesian vector network analyzer calibration algorithm", *IEEE Trans. Microwave Theory & Tech.*, vol. 57, no. 4, pp. 869-880, April 2009.
- [14] Agilent Technologies, *Specifying Calibration Standards and Kits for Agilent Vector Network Analyzers*, Application Note 1287-11, Online: <http://cp.literature.agilent.com/litweb/pdf/5989-4840EN.pdf>, March 2011.
- [15] M. Wollensack, *METAS UncLib*, Online: <http://www.metas.ch/unclib>, 2009.