METAS VNA Tools II - Math Reference V2.1

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Contents

1 Introduction

This document describes the computation of the uncertainties of coaxial S-parameter measurements, see [1]. This is a rather complicated task as all treated quantities are complex and the required operations are numerous.

Up to now a common way to handle uncertainties of vector network analyzer (VNA) measurements is the ripple technique described in [2]. The ripple technique uses precision airlines and other physical standards to extract the residual errors of a calibrated VNA system. It is based on the assumption that the precision transmission line has zero reflection. However, the systematic reflection coefficients of connectors make this assumption invalid, see [3]. This makes that the ripple method is unsuitable for computing the uncertainty of very accurate measurements. A more sophisticated computation of uncertainties has been implemented in StatistiCAL [4]. It relies on predefined uncertainties in standards and raw data. Thus it can capture certain imperfections of the VNA and the used standards. On the other hand it has neither a clear Bayesian nor frequentist concept for the imperfections of the VNA and does not provide means for producing a detailed uncertainty budget.

The present document describes in a first part the measurement model. A very well known measurement model for VNAs is described in [5]. However the model used here is slightly different. It is a N-port model and it has a more detailed uncertainty mechanism than [5]. The second part is the propagation of uncertainties through this measurement model. This methodology is described in the Guide to the Expression of Uncertainty in Measurement (GUM) [6], [7]. METAS UncLib [8], [9] is used for the linear propagation of uncertainties. The result is not only an uncertainty region but a list of uncertainty contributions with correlations. Thus the uncertainties can be propagated into eventual post-processing steps. A short description of METAS VNA Tools II can be found in [10].

2 VNA Measurement Model

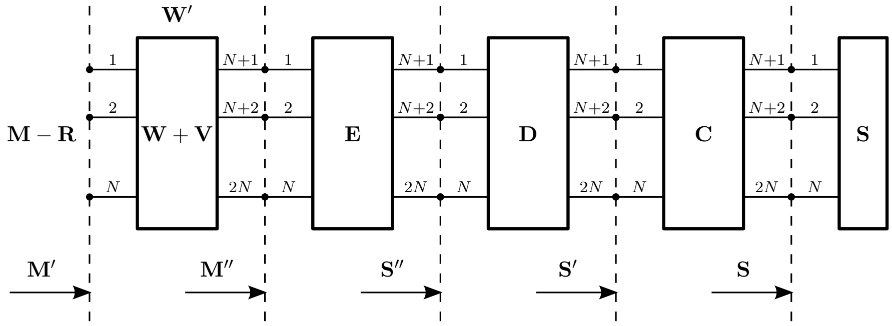

Figure 1: VNA Measurement Model

The following equation describes the in METAS VNA Tools II used N-port VNA measurement model.

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)}\right) \oplus \left[\mathbf{E} \oplus \left[\mathbf{D}^{(i)} \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)}\right]\right]\right]\right] \tag{1}$$

All bold variables are S-parameter matrices. $\mathbf{M}$, $\mathbf{R}$ and $\mathbf{S}$ are N-ports, the other bold variables are $2N$-ports and i is the measurement index.

$\mathbf{M}$ denotes the raw data measured by the VNA.

$\mathbf{R}$ denotes the noise and linearity influences.

$\mathbf{W}$ denotes the switch terms.

$\mathbf{V}$ denotes the drift of the switch terms.

$\mathbf{E}$ denotes the calibration error terms.

$\mathbf{D}$ denotes the drift of the calibration error terms.

$\mathbf{C}$ denotes the cable stability, connector repeatability and DUT uncertainty influences.

$\mathbf{S}$ denotes the error corrected data or the calibration kit standard definitions.

$\mathbf{M}$, $\mathbf{R}$, $\mathbf{V}$ and $\mathbf{D}$ change from measurement to measurement. $\mathbf{W}$ and $\mathbf{E}$ are constant during an entire calibration. $\mathbf{C}$ changes for every new connection or cable movement. $\mathbf{S}$ changes if a new device is connected.
The operator $\oplus$ denotes the cascading of two S-parameter sets, as described in appendix A.1.1.

The inverse function of equation (1) can be used for error correction.

$$\mathbf{S}^{(i)} = \left[\left[\left[\left(\mathbf{M}^{(i)} - \mathbf{R}^{(i)}\right) \ominus \left(\mathbf{W} + \mathbf{V}^{(i)}\right)\right] \ominus \mathbf{E}\right] \ominus \mathbf{D}^{(i)}\right] \ominus \mathbf{C}^{(i)} \tag{2}$$

The operator $\ominus$ denotes the decascading of two S-parameter sets, as described in appendix A.1.1.

One can introduce two new auxiliary variables.

$$\mathbf{M}'^{(i)} = \mathbf{M}^{(i)} - \mathbf{R}^{(i)} \quad (3)$$
$$\mathbf{S}'^{(i)} = \mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)} \quad (4)$$

where $\mathbf{M}'$ designates the measured data including the noise and linearity influences and $\mathbf{S}'$ designates the actual data including the cable stability and connector repeatability. Now equation (1) can be rewritten as

$$\mathbf{M}'^{(i)} = \left(\mathbf{W} + \mathbf{V}^{(i)}\right) \oplus \left[\mathbf{E} \oplus \left[\mathbf{D}^{(i)} \oplus \mathbf{S}'^{(i)}\right]\right]. \quad (5)$$

One can introduce two further auxiliary variables.

$$\mathbf{W}'^{(i)} = \mathbf{W} + \mathbf{V}^{(i)} \quad (6)$$
$$\mathbf{S}''^{(i)} = \mathbf{D}^{(i)} \oplus \mathbf{S}'^{(i)} \quad (7)$$

where $\mathbf{W}'^{(i)}$ denotes the switch terms including the drift and $\mathbf{S}''^{(i)}$ the actual data including the cable stability, connector repeatability and the drift of the calibration error terms. Now equation (5) can be rewritten as

$$\mathbf{M}'^{(i)} = \mathbf{W}'^{(i)} \oplus \left[\mathbf{E} \oplus \mathbf{S}''^{(i)}\right]. \quad (8)$$

Introducing one further auxiliary variable leads to

$$\mathbf{M}''^{(i)} = \mathbf{M}'^{(i)} \ominus \mathbf{W}'^{(i)}. \quad (9)$$

Here $\mathbf{M}''^{(i)}$ denotes the switch corrected measured data including noise, linearity influences and the drift of the switch terms. Equation (8) can be rewritten as

$$\mathbf{M}''^{(i)} = \mathbf{E} \oplus \mathbf{S}''^{(i)}. \quad (10)$$

2.1 Calibration Error Terms

The following S-parameter matrix describes the error terms of a N-port VNA.

$$\mathbf{E} = \begin{pmatrix} D_1 & X_{1,2} & & X_{1,N} & T_{1,} & & & \\ X_{2,1} & D_2 & & X_{2,N} & & T_{2,} & & \\ & & \ddots & & & & \ddots & \\ X_{N,1} & X_{N,2} & & D_N & & & & T_{N,} \\ T_{,1} & & & & M_1 & & & \\ & T_{,2} & & & & M_2 & & \\ & & \ddots & & & & \ddots & \\ & & & T_{,N} & & & & M_N \end{pmatrix} \quad (11)$$

D_x denotes the directivity of port x.

$X_{y,x}$ denotes the crosstalk from port x to port y.

$T_{x,x} = T_{x,}T_{,x}$ denotes the reflection tracking of port x.

$T_{y,x} = T_{y,}T_{,x}$ denotes the transmission tracking from port x to port y.

M_x denotes the match of port x.

And all other terms are additional crosstalk terms.

3 VNA Generic Calibration Model

The Generic VNA Model is used for the following calibration types: One Port, GSOLT, QSOLT [11], Unknown Thru [12], TRL [13], LRRM [14], Juroshek [15] and LHKM [16], [17]. If an N-port VNA has $2N$ receivers instead of $N+1$ receivers then the switch terms can be measured directly, see [18]. For one-port measurements the switch terms and the associated drift can be set to zero.

3.1 One Port Calibration

3.1.1 Reflection Error Terms

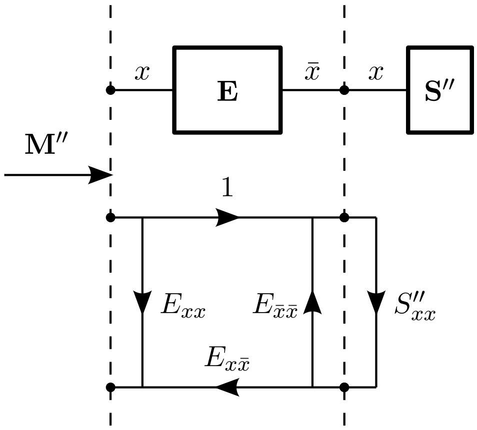

Figure 2: One Port Calibration

The following equation describes the cascading of the reflection error terms and the definition of the calibration standard

$$M''^{(i)}_{xx} = E_{xx} + \frac{E_{x\bar{x}} S''^{(i)}_{xx}}{1 - E_{\bar{x}\bar{x}} S''^{(i)}_{xx}} \tag{12}$$

with

$$\bar{x} = N + x. \tag{13}$$

N is the number of ports and x is the actual port where the reflection calibration is performed. Equation (12) can be rearranged as

$$\underbrace{M''^{(i)}_{xx}}_{y_i} = \underbrace{E_{xx}}_{p_1} + \underbrace{E_{\bar{x}\bar{x}}}_{p_2} M''^{(i)}_{xx} S''^{(i)}_{xx} + \underbrace{(E_{x\bar{x}} - E_{xx} E_{\bar{x}\bar{x}})}_{p_3} S''^{(i)}_{xx}. \tag{14}$$

Equation (14) can be written as a system of linear equations

$$\mathbf{p} = \mathbf{A}^{-1}\mathbf{y} \tag{15}$$

with

$$\mathbf{A} = \begin{pmatrix} 1 & M''^{(1)}_{xx} S''^{(1)}_{xx} & S''^{(1)}_{xx} \\ 1 & M''^{(2)}_{xx} S''^{(2)}_{xx} & S''^{(2)}_{xx} \\ 1 & M''^{(3)}_{xx} S''^{(3)}_{xx} & S''^{(3)}_{xx} \end{pmatrix} \tag{16}$$

and

$$\mathbf{y} = \begin{pmatrix} M''^{(1)}_{xx} \\ M''^{(2)}_{xx} \\ M''^{(3)}_{xx} \end{pmatrix}. \tag{17}$$

For example the first measurement could be a short, the second an open and the third a load. The vector $\mathbf{p}$ contains the solution for the error terms.

$$\begin{aligned} E_{xx} &= p_1 && (18) \\ E_{\bar{x}x} &= 1 && (19) \\ E_{x\bar{x}} &= p_3 + p_1 p_2 && (20) \\ E_{\bar{x}\bar{x}} &= p_2 && (21) \end{aligned}$$

E_{xx} stands for the directivity, $E_{x\bar{x}}E_{\bar{x}x}$ denotes the reflection tracking and $E_{\bar{x}\bar{x}}$ designates the source match term.

3.2 GSOLT Calibration

For a GSOLT calibration the switch terms have to be determined.

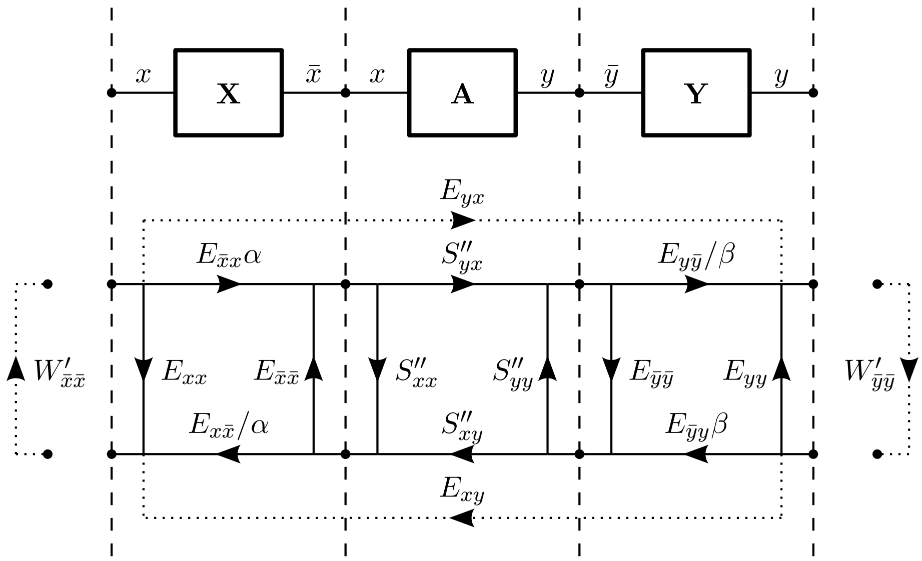

Figure 3: GSOLT Calibration

The following S-parameter matrix describes the transmission standard

$$\mathbf{A}^{(i)} = \begin{pmatrix} S''^{(i)}_{xx} & S''^{(i)}_{xy} \\ S''^{(i)}_{yx} & S''^{(i)}_{yy} \end{pmatrix}. \tag{22}$$

The error box of port x is denoted as

$$\mathbf{X} = \begin{pmatrix} E_{xx} & E_{x\bar{x}} \\ E_{\bar{x}x} & E_{\bar{x}\bar{x}} \end{pmatrix} \tag{23}$$

with

$$\bar{x} = N + x \tag{24}$$

and the error box of port y is designated by

$$\mathbf{Y} = \begin{pmatrix} E_{\bar{y}\bar{y}} & E_{\bar{y}y} \\ E_{y\bar{y}} & E_{yy} \end{pmatrix} \tag{25}$$

with

$$\bar{y} = N + y. \tag{26}$$

Cascading the error box of port x, the transmission standard definition and the error box of port y yields a new S-parameter matrix.

$$\mathbf{T}^{(i)} = \mathbf{X} \otimes \mathbf{A}^{(i)} \otimes \mathbf{Y} \tag{27}$$

The operator $\otimes$ denotes the cascading of two 2-ports, as described in appendix A.1.3.

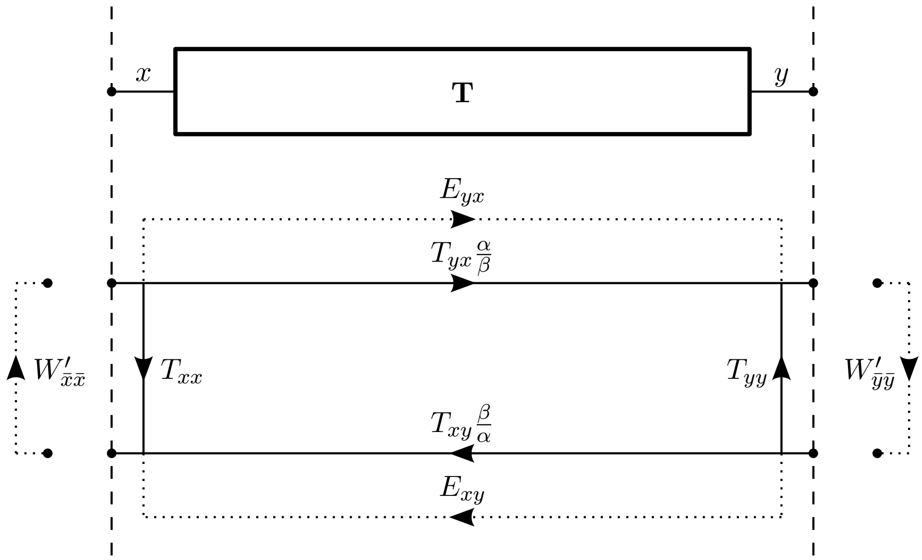

Figure 4: GSOLT Calibration (simplified)

The reflection measurement at port x is described by

$$M'^{(i)}_{xx} = T^{(i)}_{xx} + \frac{\left(\frac{\alpha^{(i)}}{\beta^{(i)}} T^{(i)}_{yx} + E_{yx}\right)\left(\frac{\beta^{(i)}}{\alpha^{(i)}} T^{(i)}_{xy} + E_{xy}\right) W'^{(i)}_{\bar{y}\bar{y}}}{1 - T^{(i)}_{yy} W'^{(i)}_{\bar{y}\bar{y}}}, \tag{28}$$

the transmission measurement from port x to y is described by

$$M'^{(i)}_{yx} = \frac{\left(\frac{\alpha^{(i)}}{\beta^{(i)}} T^{(i)}_{yx} + E_{yx}\right)}{1 - T^{(i)}_{yy} W'^{(i)}_{\bar{y}\bar{y}}}, \tag{29}$$

the transmission measurement from port y to x is described by

$$M'^{(i)}_{xy} = \frac{\left(\frac{\beta^{(i)}}{\alpha^{(i)}} T^{(i)}_{xy} + E_{xy}\right)}{1 - T^{(i)}_{xx} W'^{(i)}_{\bar{x}\bar{x}}}, \tag{30}$$

and the reflection measurement at port y is described by

$$M'^{(i)}_{yy} = T^{(i)}_{yy} + \frac{\left(\frac{\alpha^{(i)}}{\beta^{(i)}}T^{(i)}_{yx} + E_{yx}\right)\left(\frac{\beta^{(i)}}{\alpha^{(i)}}T^{(i)}_{xy} + E_{xy}\right)W'^{(i)}_{\bar{x}\bar{x}}}{1 - T^{(i)}_{xx}W'^{(i)}_{\bar{x}\bar{x}}}. \tag{31}$$

3.2.1 Switch Terms

A new auxiliary variable m_x is introduced by combining equations (28), (29) and (30)

$$m_x = \frac{M'^{(thru)}_{xx} - T^{(thru)}_{xx}}{M'^{(thru)}_{yx}M'^{(thru)}_{xy}} = \left(1 - T^{(thru)}_{xx}W'^{(thru)}_{\bar{x}\bar{x}}\right)W'^{(thru)}_{\bar{y}\bar{y}} \tag{32}$$

and m_y by combining equations (29), (30) and (31)

$$m_y = \frac{M'^{(thru)}_{yy} - T^{(thru)}_{yy}}{M'^{(thru)}_{yx}M'^{(thru)}_{xy}} = \left(1 - T^{(thru)}_{yy}W'^{(thru)}_{\bar{y}\bar{y}}\right)W'^{(thru)}_{\bar{x}\bar{x}}. \tag{33}$$

Now one has a system with two equations and two unknown variables, which are both switch terms. Equation (32) can be rewritten as

$$W'^{(thru)}_{\bar{y}\bar{y}} = \frac{m_x}{1 - T^{(thru)}_{xx}W'^{(thru)}_{\bar{x}\bar{x}}} \tag{34}$$

and one can put equation (34) into equation (33)

$$m_y = \left(1 - T^{(thru)}_{yy}\frac{m_x}{1 - T^{(thru)}_{xx}W'^{(thru)}_{\bar{x}\bar{x}}}\right)W'^{(thru)}_{\bar{x}\bar{x}}. \tag{35}$$

One can find the root of equation (35)

$$\underbrace{T^{(thru)}_{xx}}_{a}\left(W'^{(thru)}_{\bar{x}\bar{x}}\right)^2 + \underbrace{\left(m_xT^{(thru)}_{yy} - m_yT^{(thru)}_{xx} - 1\right)}_{b}W'^{(thru)}_{\bar{x}\bar{x}} + \underbrace{m_y}_{c} = 0 \tag{36}$$

and solving the quadratic equation yields the switch term of port x. The other switch term of port y can be calculated with equation (34).

$$W'^{(thru)}_{\bar{x}\bar{x}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{37}$$

Finally one can remove the drift effects from the switch terms.

$$W_{\bar{x}\bar{x}} = W'^{(thru)}_{\bar{x}\bar{x}} - V^{(thru)}_{\bar{x}\bar{x}} \tag{38}$$

$$W_{\bar{y}\bar{y}} = W'^{(thru)}_{\bar{y}\bar{y}} - V^{(thru)}_{\bar{y}\bar{y}} \tag{39}$$

3.2.2 Crosstalk

If the switch terms are known, the crosstalk between two ports can be measured directly. Note that is is assumed that there is no drift of the isolation.

$$E_{yx} = M''^{(isol)}_{yx} \tag{40}$$

$$E_{xy} = M''^{(isol)}_{xy} \tag{41}$$

3.2.3 Symmetry

The following equation defines α.

$$\alpha = E_{\bar{x}x} \tag{42}$$

To find a solution for β, equation (29) can be rewritten as

$$\beta_1 = \alpha \frac{T_{yx}^{(thru)}}{M_{yx}^{\prime(thru)}\left(1 - T_{yy}^{(thru)} W_{\bar{y}\bar{y}}^{\prime(thru)}\right) - E_{yx}} \tag{43}$$

and equation (30) can be rewritten as

$$\beta_2 = \alpha \frac{M_{xy}^{\prime(thru)}\left(1 - T_{xx}^{(thru)} W_{\bar{x}\bar{x}}^{\prime(thru)}\right) - E_{xy}}{T_{xy}^{(thru)}}. \tag{44}$$

The GSOLT calibration algorithm assumes β as the mean of the forward β_1 and the reverse β_2

$$\beta = \frac{\beta_1 + \beta_2}{2} \tag{45}$$

then one can update the reflection and transmission tracking error terms of port x and y.

$$E_{x\bar{x}} := \frac{E_{x\bar{x}} E_{\bar{x}x}}{\alpha} \tag{46}$$

$$E_{\bar{x}x} := \alpha \tag{47}$$

$$E_{y\bar{y}} := \frac{E_{y\bar{y}} E_{\bar{y}y}}{\beta} \tag{48}$$

$$E_{\bar{y}y} := \beta \tag{49}$$

$E_{x\bar{x}} E_{\bar{x}x}$ stands for the reflection tracking of port x,

$E_{y\bar{y}} E_{\bar{x}x}$ stands for the transmission tracking from port x to port y,

$E_{x\bar{x}} E_{\bar{y}y}$ stands for the transmission tracking from port y to port x and

$E_{y\bar{y}} E_{\bar{y}y}$ stands for the reflection tracking of port y.

3.3 QSOLT Calibration

The QSOLT calibration is described in [11].

3.3.1 Crosstalk

The crosstalk between two ports can be measured directly. Note that is is assumed that there is no drift of the isolation.

$$E_{yx} = M_{yx}^{\prime\prime(isol)} \tag{50}$$

$$E_{xy} = M_{xy}^{\prime\prime(isol)} \tag{51}$$

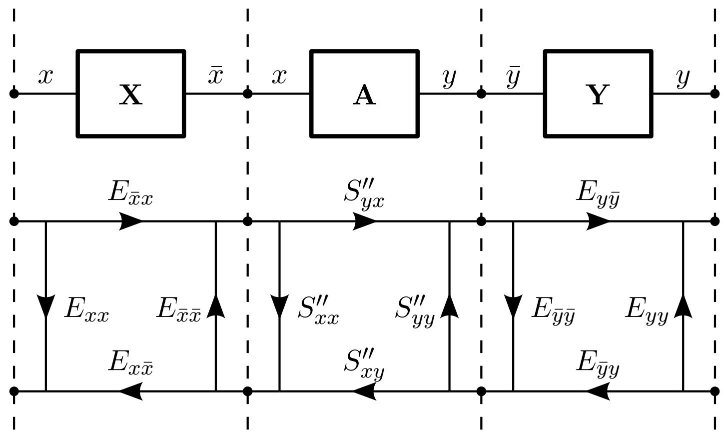

Figure 5: QSOLT Calibration

3.3.2 Copy Calibration

The following S-parameter matrix describes the switch term and crosstalk corrected measured data of the transmission standard

$$\mathbf{T}^{(i)} = \begin{pmatrix} M''^{(i)}_{xx} & M''^{(i)}_{xy} - E_{xy} \\ M''^{(i)}_{yx} - E_{yx} & M''^{(i)}_{yy} \end{pmatrix}. \tag{52}$$

Again the transmission standard is defined as

$$\mathbf{A}^{(i)} = \begin{pmatrix} S''^{(i)}_{xx} & S''^{(i)}_{xy} \\ S''^{(i)}_{yx} & S''^{(i)}_{yy} \end{pmatrix}. \tag{53}$$

The error box of port x is defined as

$$\mathbf{X} = \begin{pmatrix} E_{xx} & E_{x\bar{x}} \\ E_{\bar{x}x} & E_{\bar{x}\bar{x}} \end{pmatrix} \tag{54}$$

with

$$\bar{x} = N + x \tag{55}$$

and the error box of port y id defined as

$$\mathbf{Y} = \begin{pmatrix} E_{\bar{y}\bar{y}} & E_{\bar{y}y} \\ E_{y\bar{y}} & E_{yy} \end{pmatrix} \tag{56}$$

with

$$\bar{y} = N + y. \tag{57}$$

To copy the error terms of port x to port y one decascades $\mathbf{X}$ and $\mathbf{A}$ from $\mathbf{T}$.

$$\mathbf{Y} = \left(\mathbf{X} \otimes \mathbf{A}^{(thru)}\right)^{\ominus 1} \otimes \mathbf{T}^{(thru)} \tag{58}$$

The operator $\otimes$ denotes the cascading of two 2-ports, as described in appendix A.1.3.

3.4 Unknown Thru Calibration

The Unknown Thru calibration is described in [12].

3.4.1 Crosstalk

The crosstalk between two ports can be measured directly. Note that is is assumed that there is no drift of the isolation.

$$E_{yx} = M''^{(isol)}_{yx} \quad (59)$$
$$E_{xy} = M''^{(isol)}_{xy} \quad (60)$$

3.4.2 Symmetry

The following S-parameter matrix describes the error box of port x and y.

$$\mathbf{E}' = \begin{pmatrix} E_{xx} & E_{xy} & E_{x\bar{x}} & E_{x\bar{y}} \\ E_{yx} & E_{yy} & E_{y\bar{x}} & E_{y\bar{y}} \\ E_{\bar{x}x} & E_{\bar{x}y} & E_{\bar{x}\bar{x}} & E_{\bar{x}\bar{y}} \\ E_{\bar{y}x} & E_{\bar{y}y} & E_{\bar{y}\bar{x}} & E_{\bar{y}\bar{y}} \end{pmatrix} \quad (61)$$

One can error correct the unknown thru measurement data without knowing the symmetry error terms.

$$\mathbf{S}''^{(thru)} = \mathbf{M}''^{(thru)} \ominus \mathbf{E}' \quad (62)$$

The operator $\ominus$ denotes the decascading of two S-parameter sets, as described in appendix A.1.1.

The forward and reverse transmission S-parameter of an unknown thru have to be the same, because the unknown thru assumed to be a reciprocal device. The nominal magnitude of the transmission of the unknown thru is described by

$$|s_{yx}| = \sqrt{\left| S''^{(thru)}_{yx} S''^{(thru)}_{xy} \right|} \quad (63)$$

and the nominal phase is described by

$$\arg(s_{yx}) = \frac{\arg\left(S''^{(thru)}_{yx} S''^{(thru)}_{xy} \right)}{2}. \quad (64)$$

The following equation defines α.

$$\alpha = E_{\bar{x}x} \quad (65)$$

One can compute β with the nominal transmission and the error corrected unknown thru measurement.

$$\beta = \alpha \frac{s_{yx}}{S''^{(thru)}_{yx}} \quad (66)$$

Then one can update the reflection and transmission tracking error terms of port x and y.

$$E_{x\bar{x}} := \frac{E_{x\bar{x}}E_{\bar{x}x}}{\alpha} \quad (67)$$
$$E_{\bar{x}x} := \alpha \quad (68)$$
$$E_{y\bar{y}} := \frac{E_{y\bar{y}}E_{\bar{y}y}}{\beta} \quad (69)$$
$$E_{\bar{y}y} := \beta \quad (70)$$

$E_{x\bar{x}}E_{\bar{x}x}$ is the reflection tracking of port x,

$E_{y\bar{y}}E_{\bar{x}x}$ is the transmission tracking from port x to port y,

$E_{x\bar{x}}E_{\bar{y}y}$ is the transmission tracking from port y to port x and

$E_{y\bar{y}}E_{\bar{y}y}$ is the reflection tracking of port y.

3.5 TRL Calibration

The TRL calibration is described in [13] works only with two ports.

3.5.1 Crosstalk

The crosstalk between two ports can be measured directly. Note that is is assumed that there is no drift of the isolation.

$$E_{yx} = M''^{(isol)}_{yx} \quad (71)$$
$$E_{xy} = M''^{(isol)}_{xy} \quad (72)$$

3.5.2 Thru Reflect Line

The following T-parameter matrix describes the measurement of the thru which is corrected for switch terms and crosstalk.

$$\mathbf{T}_{thru} = \mathrm{StoTParam}\begin{pmatrix} M''^{(thru)}_{xx} & M''^{(thru)}_{xy} - E_{xy} \\ M''^{(thru)}_{yx} - E_{yx} & M''^{(thru)}_{yy} \end{pmatrix} \quad (73)$$

The next matrix describes the measurement of the line which is corrected for switch terms and crosstalk.

$$\mathbf{T}_{line} = \mathrm{StoTParam}\begin{pmatrix} M''^{(line)}_{xx} & M''^{(line)}_{xy} - E_{xy} \\ M''^{(line)}_{yx} - E_{yx} & M''^{(line)}_{yy} \end{pmatrix} \quad (74)$$

One can cascade the line and the inverted thru.

$$\mathbf{m} = \mathbf{T}_{line} \times \mathbf{T}^{-1}_{thru} \quad (75)$$

With the elements of the matrix $\mathbf{m}$ one forms

$$a_m = m_{21} \quad (76)$$
$$b_m = m_{22} - m_{11} \quad (77)$$
$$c_m = -m_{12}. \quad (78)$$

$$x_1 = \frac{-b_m + \sqrt{b_m^2 - 4a_m c_m}}{2a_m} \tag{79}$$

$$x_2 = \frac{-b_m - \sqrt{b_m^2 - 4a_m c_m}}{2a_m} \tag{80}$$

If $|x_1| > |x_2|$ then one sets

$$\alpha = x_1 \tag{81}$$

$$\beta = x_2 \tag{82}$$

else one makes the inverse assignment

$$\alpha = x_2 \tag{83}$$

$$\beta = x_1. \tag{84}$$

One can cascade the inverted thru and the line.

$$\mathbf{n} = \mathbf{T}_{thru}^{-1} \times \mathbf{T}_{line} \tag{85}$$

With the elements of the matrix $\mathbf{n}$ one forms

$$a_n = n_{12} \tag{86}$$

$$b_n = n_{22} - n_{11} \tag{87}$$

$$c_n = -n_{21}. \tag{88}$$

$$y_1 = \frac{-b_n + \sqrt{b_n^2 - 4a_n c_n}}{2a_n} \tag{89}$$

$$y_2 = \frac{-b_n - \sqrt{b_n^2 - 4a_n c_n}}{2a_n} \tag{90}$$

If $|y_1| > |y_2|$ then one sets

$$\gamma = y_1 \tag{91}$$

$$\delta = y_2 \tag{92}$$

else one makes the inverse assignment

$$\gamma = y_2 \tag{93}$$

$$\delta = y_1. \tag{94}$$

The TRL algorithm additionally uses the following intermediate quantities.

$$f_1 = \frac{\beta - M_{xx}^{\prime\prime(reflect)}}{\alpha - M_{xx}^{\prime\prime(reflect)}} \tag{95}$$

$$f_2 = \frac{\gamma + M_{yy}^{\prime\prime(reflect)}}{\delta + M_{yy}^{\prime\prime(reflect)}} \tag{96}$$

$$f_3 = \frac{\beta - M_{xx}^{\prime\prime(thru)}}{\alpha - M_{xx}^{\prime\prime(thru)}} \tag{97}$$

Now one can determine the parameters of the cascaded error boxes. The sign of e_{11} is chosen by the approximate definition of the reflection standard.

$$e_{00} = \beta \quad (98)$$
$$e_{33} = -\delta \quad (99)$$
$$e_{11} = \pm\sqrt{f_1 f_2 f_3} \quad (100)$$
$$e_{22} = \frac{f_3}{e_{11}} \quad (101)$$
$$e_{1001} = (\beta - a)\, e_{11} \quad (102)$$
$$e_{2332} = (\gamma - d)\, e_{22} \quad (103)$$
$$e_{1032} = \left(M''^{(thru)}_{yx} - E_{yx}\right)(1 - e_{11}e_{22}) \quad (104)$$
$$e_{2301} = \left(M''^{(thru)}_{xy} - E_{xy}\right)(1 - e_{11}e_{22}) \quad (105)$$

Now the offset delay $d = e^{-\gamma l}$ of the thru definition is removed from the error boxes.

$$e_{11} := e_{11}/d \quad (106)$$
$$e_{22} := e_{22}/d \quad (107)$$
$$e_{1001} := e_{1001}/d \quad (108)$$
$$e_{2332} := e_{2332}/d \quad (109)$$
$$e_{1032} := e_{1032}/d \quad (110)$$
$$e_{2301} := e_{2301}/d \quad (111)$$

Setting one transmission term to 1 defines the remaining terms.

$$e_{10} = 1 \quad (112)$$
$$e_{01} = \frac{e_{1001}}{e_{10}} \quad (113)$$
$$e_{32} = \frac{e_{1032}}{e_{10}} \quad (114)$$
$$e_{23} = \frac{e_{2301}}{e_{01}} \quad (115)$$

Finally one can update the error terms of the VNA measurement model.

$$E_{xx} = e_{00} \quad (116)$$
$$E_{\bar{x}x} = e_{10} \quad (117)$$
$$E_{x\bar{x}} = e_{01} \quad (118)$$
$$E_{\bar{x}\bar{x}} = e_{11} \quad (119)$$
$$E_{\bar{y}\bar{y}} = e_{22} \quad (120)$$
$$E_{y\bar{y}} = e_{32} \quad (121)$$
$$E_{\bar{y}y} = e_{23} \quad (122)$$
$$E_{yy} = e_{33} \quad (123)$$

E_{xx} is the directivity of port x,

$E_{\bar{x}\bar{x}}$ is the match of port x,

$E_{x\bar{x}}E_{\bar{x}x}$ is the reflection tracking of port x,

$E_{y\bar{y}}E_{\bar{x}x}$ is the transmission tracking from port x to port y,

$E_{x\bar{x}}E_{\bar{y}y}$ is the transmission tracking from port y to port x,

$E_{y\bar{y}}E_{\bar{y}y}$ is the reflection tracking of port y,

$E_{\bar{y}\bar{y}}$ is the match of port y and

E_{yy} is the directivity of port y.

3.6 LRRM Calibration

The in METAS VNA Tools II implemented LRRM algorithm is a generalization of the LRRM calibration described in [14]. The generalization consists of implementing a reflective non-reciprocal line standard. The line standard has to be fully known. The two reflection standards are measured each at both ports and have to have different reflection angles or amplitudes. The match standard is only measured at one port. The DC resistance R of the match has to be known. The following table describes the used calibration standards in the LRRM calibration.

Table 1: LRRM Standards

Standard	S-parameter
Line	$\mathbf{S}^{(t)} = \begin{pmatrix} s_{xx}^{(t)} & s_{xy}^{(t)} \\ s_{yx}^{(t)} & s_{yy}^{(t)} \end{pmatrix}$
Reflect 1 (unknown capacitance)	$S^{(o)} = \frac{1-j\omega C Z_r}{1+j\omega C Z_r}$ with $C = ?$ and $C \geq 0$
Reflect 2 (unknown reflection)	$S^{(r)} = r_{re} + j r_{im}$ with $r_{re} = ?$ and $r_{im} = ?$
Match (unknown inductance)	$S^{(m)} = \frac{R+j\omega L-Z_r}{R+j\omega L+Z_r}$ with $L = ?$

The LRRM calibration can be described by the following nine linear equations, see section 3.8.

$$\left(\begin{array}{cccccccc}
M''^{(t)}_{xx} & 0 & 1 & 0 & M''^{(t)}_{xx}S''^{(t)}_{xx} & M''^{(t)}_{xy}S''^{(t)}_{yx} & S''^{(t)}_{xx} & 0 \\
M''^{(t)}_{yx} & 0 & 0 & 0 & M''^{(t)}_{yx}S''^{(t)}_{xx} & M''^{(t)}_{yy}S''^{(t)}_{yx} & 0 & S''^{(t)}_{yx} \\
0 & M''^{(t)}_{xy} & 0 & 0 & M''^{(t)}_{xx}S''^{(t)}_{xy} & M''^{(t)}_{xy}S''^{(t)}_{yy} & S''^{(t)}_{xy} & 0 \\
0 & M''^{(t)}_{yy} & 0 & 1 & M''^{(t)}_{yx}S''^{(t)}_{xy} & M''^{(t)}_{yy}S''^{(t)}_{yy} & 0 & S''^{(t)}_{yy} \\
\hline
M''^{(o)}_{xx} & 0 & 1 & 0 & M''^{(o)}_{xx}S''^{(o)}_{xx} & 0 & S''^{(o)}_{xx} & 0 \\
0 & M''^{(o)}_{yy} & 0 & 1 & 0 & M''^{(o)}_{yy}S''^{(o)}_{yy} & 0 & S''^{(o)}_{yy} \\
\hline
M''^{(r)}_{xx} & 0 & 1 & 0 & M''^{(r)}_{xx}S''^{(r)}_{xx} & 0 & S''^{(r)}_{xx} & 0 \\
0 & M''^{(r)}_{yy} & 0 & 1 & 0 & M''^{(r)}_{yy}S''^{(r)}_{yy} & 0 & S''^{(r)}_{yy} \\
\hline
M''^{(m)}_{xx} & 0 & 1 & 0 & M''^{(m)}_{xx}S''^{(m)}_{xx} & 0 & S''^{(m)}_{xx} & 0
\end{array}\right)
\begin{pmatrix} 1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix} = 0 \qquad (124)$$

The LRRM calibration is solved with the following steps:

1. The seven unknowns x_2 to x_7 which represent the error terms can be eliminated. This yields two linear equations.
2. The numerators of these two equations have to be equal to zero:

$$\begin{aligned}
&M''^{(o)}_{xx}((M''^{(o)}_{yy} - M''^{(t)}_{yy})(M''^{(r)}_{xx}(S''^{(o)}_{xx} - S''^{(r)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))\\
&\quad - M''^{(t)}_{xx}(S''^{(m)}_{xx} - S''^{(r)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy})) + M''^{(m)}_{xx}(S''^{(m)}_{xx}\\
&\quad - S''^{(o)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(r)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))) - M''^{(t)}_{xy}M''^{(t)}_{yx}(S''^{(m)}_{xx}\\
&\quad - S''^{(r)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))) + M''^{(r)}_{xx}(-(M''^{(o)}_{yy}\\
&\quad - M''^{(t)}_{yy})(M''^{(m)}_{xx}(S''^{(m)}_{xx} - S''^{(r)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))\\
&\quad - M''^{(t)}_{xx}(S''^{(m)}_{xx} - S''^{(o)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(r)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy})))\\
&\quad + M''^{(t)}_{xy}M''^{(t)}_{yx}(S''^{(m)}_{xx} - S''^{(o)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(r)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy})))\\
&\quad + M''^{(m)}_{xx}(M''^{(t)}_{xy}M''^{(t)}_{yx} + M''^{(t)}_{xx}(M''^{(o)}_{yy} - M''^{(t)}_{yy}))(S''^{(o)}_{xx} - S''^{(r)}_{xx})(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx}\\
&\quad - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy})) = 0
\end{aligned} \tag{125}$$

and

$$\begin{aligned}
&(M''^{(t)}_{xy}M''^{(t)}_{yx} + M''^{(t)}_{xx}(M''^{(o)}_{yy} - M''^{(t)}_{yy}))(M''^{(t)}_{xy}M''^{(t)}_{yx} + M''^{(t)}_{xx}(M''^{(r)}_{yy} - M''^{(t)}_{yy}))(S''^{(m)}_{xx}\\
&\quad - S''^{(o)}_{xx})S''^{(t)}_{xy}S''^{(t)}_{yx}(S''^{(o)}_{yy} - S''^{(r)}_{yy}) + M''^{(o)}_{xx}(M''^{(t)}_{xx}(M''^{(o)}_{yy} - M''^{(t)}_{yy})(-M''^{(r)}_{yy}\\
&\quad + M''^{(t)}_{yy})(S''^{(m)}_{xx} - S''^{(o)}_{xx})S''^{(t)}_{xy}S''^{(t)}_{yx}(S''^{(o)}_{yy} - S''^{(r)}_{yy}) + M''^{(t)}_{xy}M''^{(t)}_{yx}(M''^{(t)}_{yy}(S''^{(m)}_{xx}\\
&\quad - S''^{(o)}_{xx})S''^{(t)}_{xy}S''^{(t)}_{yx}(S''^{(o)}_{yy} - S''^{(r)}_{yy}) - M''^{(r)}_{yy}(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy}\\
&\quad - S''^{(t)}_{yy}))(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx} - S''^{(t)}_{xx})(S''^{(r)}_{yy} - S''^{(t)}_{yy})) + M''^{(o)}_{yy}(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx}\\
&\quad - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(r)}_{yy} - S''^{(t)}_{yy})))) + M''^{(m)}_{xx}((M''^{(o)}_{xx}\\
&\quad - M''^{(t)}_{xx})(M''^{(o)}_{yy} - M''^{(t)}_{yy})(M''^{(r)}_{yy} - M''^{(t)}_{yy})(S''^{(m)}_{xx} - S''^{(o)}_{xx})S''^{(t)}_{xy}S''^{(t)}_{yx}(S''^{(o)}_{yy} - S''^{(r)}_{yy})\\
&\quad + M''^{(t)}_{xy}M''^{(t)}_{yx}(M''^{(t)}_{yy}(S''^{(m)}_{xx} - S''^{(o)}_{xx})S''^{(t)}_{xy}S''^{(t)}_{yx}(S''^{(o)}_{yy} - S''^{(r)}_{yy}) - M''^{(o)}_{yy}(S''^{(t)}_{xy}S''^{(t)}_{yx}\\
&\quad - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx} - S''^{(t)}_{xx})(S''^{(r)}_{yy} - S''^{(t)}_{yy}))\\
&\quad + M''^{(r)}_{yy}(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(m)}_{xx} - S''^{(t)}_{xx})(S''^{(o)}_{yy} - S''^{(t)}_{yy}))(S''^{(t)}_{xy}S''^{(t)}_{yx} - (S''^{(o)}_{xx} - S''^{(t)}_{xx})(S''^{(r)}_{yy}\\
&\quad - S''^{(t)}_{yy})))) = 0.
\end{aligned} \tag{126}$$

3. $S''^{(o)}_{xx}$ and $S''^{(o)}_{yy}$ can be replaced by the unknown capacitance C:

$$S''^{(o)}_{xx} = D^{(o)}_{xx} + \frac{D^{(o)}_{\bar{x}x} S'^{(o)}_{xx} D^{(o)}_{x\bar{x}}}{1 - D^{(o)}_{\bar{x}\bar{x}} S'^{(o)}_{xx}} \text{ with } S'^{(o)}_{xx} = C^{(o)}_{xx} + \frac{C^{(o)}_{\bar{x}x} S^{(o)} C^{(o)}_{x\bar{x}}}{1 - C^{(o)}_{\bar{x}\bar{x}} S^{(o)}} \tag{127}$$

and

$$S''^{(o)}_{yy} = D^{(o)}_{yy} + \frac{D^{(o)}_{\bar{y}y} S'^{(o)}_{yy} D^{(o)}_{y\bar{y}}}{1 - D^{(o)}_{\bar{y}\bar{y}} S'^{(o)}_{yy}} \text{ with } S'^{(o)}_{yy} = C^{(o)}_{yy} + \frac{C^{(o)}_{\bar{y}y} S^{(o)} C^{(o)}_{y\bar{y}}}{1 - C^{(o)}_{\bar{y}\bar{y}} S^{(o)}} \tag{128}$$

where

$$S^{(o)} = \frac{1 - j\omega C Z_r}{1 + j\omega C Z_r}. \tag{129}$$

Here $C^{(o)}_{xx}$, $C^{(o)}_{\bar{x}x}$, $C^{(o)}_{x\bar{x}}$ and $C^{(o)}_{\bar{x}\bar{x}}$ are cable influences and $D^{(o)}_{xx}$, $D^{(o)}_{\bar{x}x}$, $D^{(o)}_{x\bar{x}}$ and $D^{(o)}_{\bar{x}\bar{x}}$ are drift influences of the reflection standard 1 (open) measurement at port x.

$C^{(o)}_{yy}$, $C^{(o)}_{\bar{y}y}$, $C^{(o)}_{y\bar{y}}$ and $C^{(o)}_{\bar{y}\bar{y}}$ are cable influences and $D^{(o)}_{yy}$, $D^{(o)}_{\bar{y}y}$, $D^{(o)}_{y\bar{y}}$ and $D^{(o)}_{\bar{y}\bar{y}}$ are drift influences of the reflection standard 1 (open) measurement at port y.

4. $S''^{(r)}_{xx}$ and $S''^{(r)}_{yy}$ can be replaced by the unknown reflection r_{re} and r_{im}:

$$S''^{(r)}_{xx} = D^{(r)}_{xx} + \frac{D^{(r)}_{\bar{x}x} S'^{(r)}_{xx} D^{(r)}_{x\bar{x}}}{1 - D^{(r)}_{\bar{x}\bar{x}} S'^{(r)}_{xx}} \text{ with } S'^{(r)}_{xx} = C^{(r)}_{xx} + \frac{C^{(r)}_{\bar{x}x} S^{(r)} C^{(r)}_{x\bar{x}}}{1 - C^{(r)}_{\bar{x}\bar{x}} S^{(r)}} \tag{130}$$

and

$$S''^{(r)}_{yy} = D^{(r)}_{yy} + \frac{D^{(r)}_{\bar{y}y} S'^{(r)}_{yy} D^{(r)}_{y\bar{y}}}{1 - D^{(r)}_{\bar{y}\bar{y}} S'^{(r)}_{yy}} \text{ with } S'^{(r)}_{yy} = C^{(r)}_{yy} + \frac{C^{(r)}_{\bar{y}y} S^{(r)} C^{(r)}_{y\bar{y}}}{1 - C^{(r)}_{\bar{y}\bar{y}} S^{(r)}} \tag{131}$$

where

$$S^{(r)} = r_{re} + j r_{im}. \tag{132}$$

Here $C^{(r)}_{xx}$, $C^{(r)}_{\bar{x}x}$, $C^{(r)}_{x\bar{x}}$ and $C^{(r)}_{\bar{x}\bar{x}}$ are cable influences and $D^{(r)}_{xx}$, $D^{(r)}_{\bar{x}x}$, $D^{(r)}_{x\bar{x}}$ and $D^{(r)}_{\bar{x}\bar{x}}$ are drift influences of the reflection standard 2 measurement at port x.

$C^{(r)}_{yy}$, $C^{(r)}_{\bar{y}y}$, $C^{(r)}_{y\bar{y}}$ and $C^{(r)}_{\bar{y}\bar{y}}$ are cable influences and $D^{(r)}_{yy}$, $D^{(r)}_{\bar{y}y}$, $D^{(r)}_{y\bar{y}}$ and $D^{(r)}_{\bar{y}\bar{y}}$ are drift influences of the reflection standard 2 measurement at port y.

5. $S''^{(m)}_{xx}$ can be replaced by the unknown series inductance L:

$$S''^{(m)}_{xx} = D^{(m)}_{xx} + \frac{D^{(m)}_{\bar{x}x} S'^{(m)}_{xx} D^{(m)}_{x\bar{x}}}{1 - D^{(m)}_{\bar{x}\bar{x}} S'^{(m)}_{xx}} \text{ with } S'^{(m)}_{xx} = C^{(m)}_{xx} + \frac{C^{(m)}_{\bar{x}x} S^{(m)} C^{(m)}_{x\bar{x}}}{1 - C^{(m)}_{\bar{x}\bar{x}} S^{(m)}} \tag{133}$$

and

$$S^{(m)} = \frac{R + j\omega L - Z_r}{R + j\omega L + Z_r}. \tag{134}$$

Here $C^{(m)}_{xx}$, $C^{(m)}_{\bar{x}x}$, $C^{(m)}_{x\bar{x}}$ and $C^{(m)}_{\bar{x}\bar{x}}$ are cable influences and $D^{(m)}_{xx}$, $D^{(m)}_{\bar{x}x}$, $D^{(m)}_{x\bar{x}}$ and $D^{(m)}_{\bar{x}\bar{x}}$ are drift influences of the match measurement at port x.

6. This finally yields two complex non-linear equations with four scalar unknowns.

7. Non-linear optimization yields C, r_{re}, r_{im} and L. This optimization is independent of the error terms. The starting values are $C = 0$, $r_{re} = -1$, $r_{im} = 0$ and $L = 0$.

8. Now all standards are fully known and the QSOLT calibration, see section 3.3, is used to compute the error terms.

3.7 Juroshek Calibration

The Juroshek calibration is described in [15]. The following assignment prepares the raw data measured by the VNA.

$$M'^{(i)}_{xx} := \frac{M'^{(i)}_{xx}}{M'^{(i)}_{yx}} \tag{135}$$

Finally one can compute a one port calibration, see section 3.1. Where

x is the VNA port where port 1 of the splitter is connected,

y is the VNA port where port 2 or 3 of the splitter is connected,

$E_{\bar{x}\bar{x}}$ is the equivalent source match of port 3 or 2 of the splitter.

3.8 LHKM Calibration

The LHKM calibration is described in [16], [17]. This calibration is not fully implemented in METAS VNA Tools II and it's still under development.

The following equation describes the in METAS VNA Tools II used LHKM calibration model

$$\mathbf{M}''^{(i)}\mathbf{A} - \mathbf{B} - \mathbf{M}''^{(i)}\mathbf{C}\mathbf{S}''^{(i)} + \mathbf{D}\mathbf{S}''^{(i)} \tag{136}$$

with

$$\mathbf{A}_{00} = 1 \tag{137}$$

where $\mathbf{M}''$ denotes the switch corrected measured data including noise and linearity influences. And $\mathbf{S}''$ denotes the actual data including the cable stability, connector repeatability and the drift of the calibration error terms.

The ABCD terms of the LHKM calibration model can be converted to the generic calibration model. The result will be the error terms $\mathbf{E}$

$$\mathbf{E} = \begin{pmatrix} \mathbf{E}_{00} & \mathbf{E}_{01} \\ \mathbf{E}_{10} & \mathbf{E}_{11} \end{pmatrix} \tag{138}$$

with

$$\mathbf{E}_{10} = \mathbf{A}^{-1} \tag{139}$$

$$\mathbf{E}_{00} = \mathbf{B}\mathbf{A}^{-1} \tag{140}$$

$$\mathbf{E}_{11} = \mathbf{A}^{-1}\mathbf{C} \tag{141}$$

$$\mathbf{E}_{01} = \mathbf{B}\mathbf{A}^{-1}\mathbf{C} - \mathbf{D}. \tag{142}$$

3.8.1 TRL, LRL, TRM, LRM

The LHKM (TRL, LRL) and LHKM (TRM, LRM) calibrations are described in [17]. These calibration algorithms and the associated uncertainty propagation can lead to over-determined linear and quadratic eigenvalue problems. The over-determined non-linear eigenvalue problem is described in appendix E.

4 VNA Switched Calibration Model

The Switched VNA Model uses N error terms matrices $\overset{x}{\mathbf{E}}$ for an N-port VNA. One for each switch position x of the source. The switch terms matrix $\mathbf{W}$ and the associated drift $\mathbf{V}$ is set to zero. The Switched VNA Model is used for the SOLT calibration.

4.1 SOLT Calibration

4.1.1 Reflection

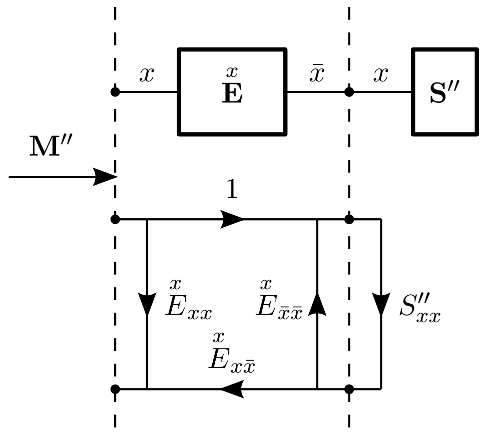

Figure 6: SOLT Reflection Calibration

The following equation describes the cascading of the reflection error terms and the definition of the calibration standard

$$M''^{(i)}_{xx} = \overset{x}{E}_{xx} + \frac{\overset{x}{E}_{x\bar{x}} + S''^{(i)}_{xx}}{1 - \overset{x}{E}_{\bar{x}\bar{x}} S''^{(i)}_{xx}} \tag{143}$$

with

$$\bar{x} = N + x. \tag{144}$$

N is the number of ports and x is the actual port where the reflection calibration is performed. Equation (143) can be rearranged as

$$\underbrace{M''^{(i)}_{xx}}_{y_i} = \underbrace{\overset{x}{E}_{xx}}_{p_1} + \underbrace{\overset{x}{E}_{\bar{x}\bar{x}}}_{p_2} M''^{(i)}_{xx} S''^{(i)}_{xx} + \underbrace{\left(\overset{x}{E}_{x\bar{x}} - \overset{x}{E}_{xx} \overset{x}{E}_{\bar{x}\bar{x}} \right)}_{p_3} S''^{(i)}_{xx}. \tag{145}$$

Equation (145) can be written as a system of linear equations

$$\mathbf{p} = \mathbf{A}^{-1}\mathbf{y} \tag{146}$$

with

$$\mathbf{A} = \begin{pmatrix} 1 & M''^{(1)}_{xx} S''^{(1)}_{xx} & S''^{(1)}_{xx} \\ 1 & M''^{(2)}_{xx} S''^{(2)}_{xx} & S''^{(2)}_{xx} \\ 1 & M''^{(3)}_{xx} S''^{(3)}_{xx} & S''^{(3)}_{xx} \end{pmatrix} \tag{147}$$

and

$$\mathbf{y} = \begin{pmatrix} M''^{(1)}_{xx} \\ M''^{(2)}_{xx} \\ M''^{(3)}_{xx} \end{pmatrix}. \tag{148}$$

For example the first measurement could be a short, the second an open and the third a load. The vector $\mathbf{p}$ contains the solution for the error terms.

$$\overset{x}{E}_{xx} = p_1 \tag{149}$$
$$\overset{x}{E}_{\bar{x}x} = 1 \tag{150}$$
$$\overset{x}{E}_{x\bar{x}} = p_3 + p_1 p_2 \tag{151}$$
$$\overset{x}{E}_{\bar{x}\bar{x}} = p_2 \tag{152}$$

$\overset{x}{E}_{xx}$ stands for the directivity, $\overset{x}{E}_{x\bar{x}}\overset{x}{E}_{\bar{x}x}$ denotes the reflection tracking and $\overset{x}{E}_{\bar{x}\bar{x}}$ designates the source match term.

4.1.2 Isolation

The isolation between two ports can be measured directly. Note that is is assumed that there is no drift of the isolation.

$$\overset{x}{E}_{yx} = M''^{(isol)}_{yx} \tag{153}$$
$$\overset{y}{E}_{xy} = M''^{(isol)}_{xy} \tag{154}$$

4.1.3 Transmission

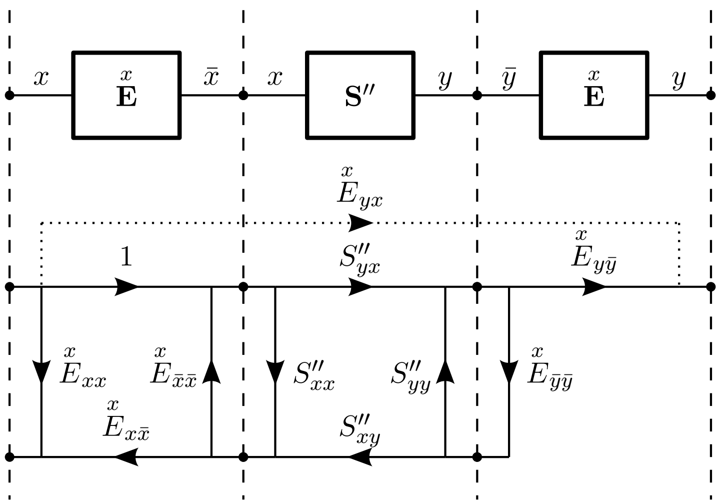

Figure 7: SOLT Forward Transmission Calibration

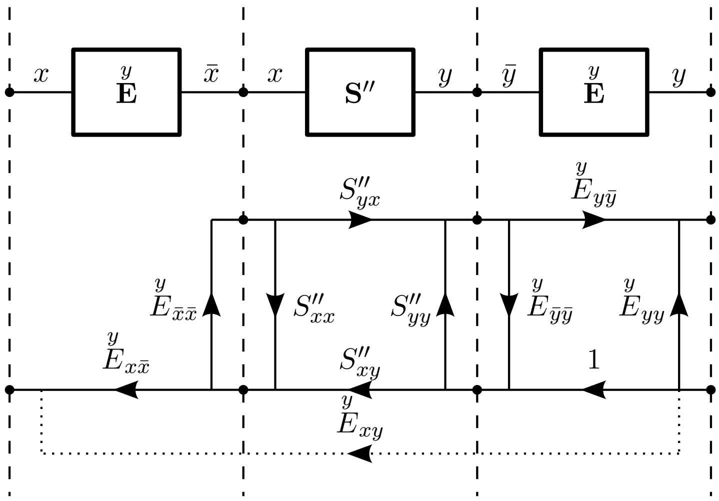

Figure 8: SOLT Reverse Transmission Calibration

The following S-parameter matrix describes the transmission standard

$$\mathbf{A}^{(i)} = \begin{pmatrix} S''^{(i)}_{xx} & S''^{(i)}_{xy} \\ S''^{(i)}_{yx} & S''^{(i)}_{yy} \end{pmatrix}. \tag{155}$$

The error box of port x is denoted as

$$\mathbf{X} = \begin{pmatrix} \overset{x}{E}_{xx} & \overset{x}{E}_{x\bar{x}} \\ \overset{x}{E}_{\bar{x}x} & \overset{x}{E}_{\bar{x}\bar{x}} \end{pmatrix} \tag{156}$$

with

$$\bar{x} = N + x \tag{157}$$

and the error box of port y is denoted as

$$\mathbf{Y} = \begin{pmatrix} \overset{y}{E}_{\bar{y}\bar{y}} & \overset{y}{E}_{\bar{y}y} \\ \overset{y}{E}_{y\bar{y}} & \overset{y}{E}_{yy} \end{pmatrix} \tag{158}$$

with

$$\bar{y} = N + y. \tag{159}$$

Cascading the error box of port x and the thru definition yields a new S-parameter matrix

$$\overset{x}{\mathbf{T}}^{(i)} = \mathbf{X} \otimes \mathbf{A}^{(i)}. \tag{160}$$

The same can be done for port y

$$\overset{y}{\mathbf{T}}^{(i)} = \mathbf{A}^{(i)} \otimes \mathbf{Y}. \tag{161}$$

The operator $\otimes$ denotes the cascading of two 2-ports, as described in appendix A.1.3.

One can introduce four new auxiliary variables, which describe the directivity and isolation corrected measurement of the transmission standard.

$$s_{xx} = M''^{(thru)}_{xx} - \overset{x}{T}^{(thru)}_{xx} \tag{162}$$
$$s_{yx} = M''^{(thru)}_{yx} - \overset{x}{E}_{yx} \tag{163}$$
$$s_{xy} = M''^{(thru)}_{xy} - \overset{y}{E}_{xy} \tag{164}$$
$$s_{yy} = M''^{(thru)}_{yy} - \overset{y}{T}^{(thru)}_{yy} \tag{165}$$

Next one can come up with equations for the transmission tracking and the load match of port x and y.

$$\overset{x}{E}_{\bar{y}\bar{y}} = \frac{s_{xx}}{s_{xx}\overset{x}{T}^{(thru)}_{yy} + \overset{x}{T}^{(thru)}_{yx}\overset{x}{T}^{(thru)}_{xy}} \tag{166}$$
$$\overset{x}{E}_{y\bar{y}} = s_{yx}\frac{1 - \overset{x}{E}_{\bar{y}\bar{y}}\overset{x}{T}^{(thru)}_{yy}}{\overset{x}{T}^{(thru)}_{yx}} \tag{167}$$
$$\overset{y}{E}_{\bar{x}\bar{x}} = \frac{s_{yy}}{s_{yy}\overset{y}{T}^{(thru)}_{xx} + \overset{y}{T}^{(thru)}_{yx}\overset{y}{T}^{(thru)}_{xy}} \tag{168}$$
$$\overset{y}{E}_{x\bar{x}} = s_{xy}\frac{1 - \overset{y}{E}_{\bar{x}\bar{x}}\overset{y}{T}^{(thru)}_{yy}}{\overset{y}{T}^{(thru)}_{xy}} \tag{169}$$

$\overset{x}{E}_{\bar{y}\bar{y}}$ is the load match of port y,

$\overset{x}{E}_{y\bar{y}}$ is the transmission tracking from port x to port y,

$\overset{y}{E}_{x\bar{x}}$ is the transmission tracking from port y to port x and

$\overset{y}{E}_{\bar{x}\bar{x}}$ is the load match of port x.

5 VNA Optimization Calibration

For the optimization calibration [19] the optimizer computes the switch and calibration error terms and the unknown terms of the calibration standard definitions for an over-determined calibration. It uses an optimization algorithm for the VNA measurement model. The following equation describes the in METAS VNA Tools II used N-port VNA measurement model, see section 2.

$$\mathbf{M}^{(i)} = \mathbf{R}^{(i)} + \left[\left(\mathbf{W} + \mathbf{V}^{(i)}\right) \oplus \left[\mathbf{E} \oplus \left[\mathbf{D}^{(i)} \oplus \left[\mathbf{C}^{(i)} \oplus \mathbf{S}^{(i)}\right]\right]\right]\right] \tag{170}$$

The inverse function of the above equation can be used for error correction.

$$\mathbf{S}^{(i)} = \left[\left[\left[\left(\mathbf{M}^{(i)} - \mathbf{R}^{(i)}\right) \ominus \left(\mathbf{W} + \mathbf{V}^{(i)}\right)\right] \ominus \mathbf{E}\right] \ominus \mathbf{D}^{(i)}\right] \ominus \mathbf{C}^{(i)} \tag{171}$$

The optimizer minimizes the following objective function for all measurements.

$$\left[\left[\left[\left[\left(\mathbf{M}^{(i)} - \mathbf{R}^{(i)}\right) \ominus \left(\mathbf{W} + \mathbf{V}^{(i)}\right)\right] \ominus \mathbf{E}\right] \ominus \mathbf{D}^{(i)}\right] \ominus \mathbf{C}^{(i)}\right] - \mathbf{S}^{(i)} \tag{172}$$

5.1 Weighting

The following equation describes the objective function f where $\mathbf{X}$ are the variable optimization parameters and $\mathbf{P}$ are the constant optimization parameters.

$$\mathbf{F} = f(\mathbf{X}, \mathbf{P}) \tag{173}$$

5.1.1 Covariance Weighting

For the weighting of the optimization problem the covariance of the objective function can be used.

$$\mathbf{C}_F = \mathbf{J}_{F,P}\mathbf{C}_P\mathbf{J}'_{F,P} \tag{174}$$

The optimization problem is described with the following expression.

$$\min_{\mathbf{X}\in\mathbb{R}^n} \left(\mathbf{F}\mathbf{C}_F^{-1}\mathbf{F}'\right) \tag{175}$$

One can introduce $\mathbf{G} = \mathbf{F}\mathbf{W}_F$, then the optimization problem becomes

$$\min_{\mathbf{X}\in\mathbb{R}^n} \left(\mathbf{G}\mathbf{G}'\right) \tag{176}$$

where the weights $\mathbf{W}_F$ are computed from the covariance of the objective function $\mathbf{C}_F$ using the eigenvalue decomposition.

5.1.2 User-Defined Weighting

Using user-defined weights $\mathbf{W}_U$ changes the objective function to

$$\mathbf{G} = \mathbf{F}\mathbf{W}_U. \tag{177}$$

5.2 Uncertainty Propagation

The Jacobi matrix $\mathbf{X}$ to $\mathbf{P}$ at the point of the solution is described with the following equation.

$$\mathbf{J}_{X,P} = \left(\mathbf{J}'_{G,X}\mathbf{J}_{G,X}\right)^{-1}\mathbf{J}'_{G,X}\mathbf{J}_{G,P} \tag{178}$$

It can be used for the uncertainty propagation.

6 VNA Calibration Standard

6.1 Agilent Model Standard

The Agilent model standard is described in [20].

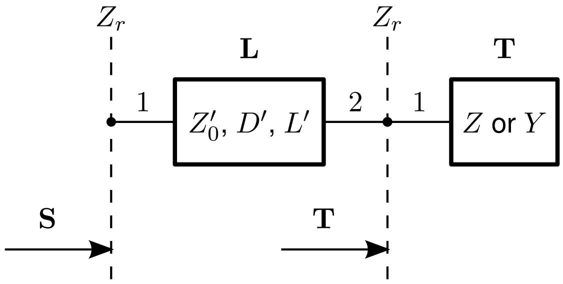

Figure 9: Agilent Model Standard

The S-parameters of an Agilent model reflection standard are defined by the following equation where $\mathbf{L}$ is the transmission line section and $\mathbf{T}$ is the reflection part.

$$\mathbf{S} = \mathbf{L} \oplus \mathbf{T} \tag{179}$$

The coaxial transmission line section is computed with the following two equations. Where Z'_0 is the Offset Z0 in Ohm, D' is the Offset Delay in s and L' is the Offset Loss in $\mathrm{Ohm/s}$.

$$Z_0 = Z'_0 \left(1 + (1-j) \frac{L'}{2\omega Z'_0} \sqrt{\frac{f}{1\,\mathrm{GHz}}} \right) \tag{180}$$

$$\gamma l = j\omega D' \left(1 + (1-j) \frac{L'}{2\omega Z'_0} \sqrt{\frac{f}{1\,\mathrm{GHz}}} \right) \tag{181}$$

The waveguide transmission line section is computed with the following equation. Where μ_0 is the vacuum permeability, ϵ_0 is the vacuum permittivity, f_c is the cutoff frequency and h/w is the height to width ratio.

$$\gamma l = D' \left(L' \sqrt{\frac{\epsilon_0}{\mu_0}} \sqrt{\frac{f}{f_c}} \left(\frac{1 + \frac{2h}{w} \left(\frac{f_c}{f} \right)^2}{\sqrt{1 - \left(\frac{f_c}{f} \right)^2}} \right) + j2\pi f \sqrt{1 - \left(\frac{f_c}{f} \right)^2} \right) \tag{182}$$

For computing the S-parameters of a transmission line section see section A.2.2. Where Z_r is the reference impedance, Z_0 is the characteristic impedance and γl is the propagation constant times the length.

6.1.1 Short

The reflection part of a short standard is defined by the following equations.

$$L_{eff} = L_0 + L_1 f + L_2 f^2 + L_3 f^3 \tag{183}$$

$$Z = j\omega L_{eff} \tag{184}$$

$$T_{11} = \frac{Z - Z_r}{Z + Z_r} \tag{185}$$

For an offset short the transmission line section is cascaded to the reflection part.

6.1.2 Open

The reflection part of a open standard is defined by the following equations.

$$C_{eff} = C_0 + C_1 f + C_2 f^2 + C_3 f^3 \tag{186}$$

$$Y = j\omega C_{eff} \tag{187}$$

$$T_{11} = \frac{1 - Z_r Y}{1 + Z_r Y} \tag{188}$$

For an offset open the transmission line section is cascaded to the reflection part.

6.1.3 Load

In the Agilent model a load has no reflection.

$$T_{11} = 0 \tag{189}$$

6.1.4 Delay / Thru

The S-parameters of a Delay / Thru are equal to the S-parameters of the line section.

$$\mathbf{S} = \mathbf{L} \tag{190}$$

6.2 Anritsu and Rohde Schwarz Model Standard

These model standards are similar to the Agilent model standard except the line section. The Offset Z0 Z_0' is set to the reference impedance.

The Offset Length D'' is defined in m.

$$D'' = \frac{D' c}{\sqrt{\varepsilon_r}} \tag{191}$$

Where D' is the Offset Delay in s, $c = 299792458\,\mathrm{m/s}$ is the speed of light and $\varepsilon_r = 1$ is the relative permittivity.

The Offset Loss L'' is defined in $\mathrm{dB}/\sqrt{\mathrm{GHz}}$.

$$L'' = 8.6859 \frac{D' L'}{Z_0'} \tag{192}$$

Where Z_0' is the Offset Z0 in Ohm, D' is the Offset Delay in s and L' is the Offset Loss in $\mathrm{Ohm/s}$.

6.3 Databased Standard

The S-parameters of a databased standard are explicitly stated for each data point.

6.4 Ideal Standard

Table 2 shows the S-parameters for ideal standards.

Table 2: Ideal Standards

Standard	S-parameter
Ideal Short	$\mathbf{S} = \begin{pmatrix} -1 \end{pmatrix}$
Ideal Open	$\mathbf{S} = \begin{pmatrix} 1 \end{pmatrix}$
Ideal Load	$\mathbf{S} = \begin{pmatrix} 0 \end{pmatrix}$
Ideal Thru	$\mathbf{S} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Ideal Isolation	$\mathbf{S} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

6.5 Unknown Standard

Table 3 shows the S-parameters for unknown standards.

Table 3: Unknown Standards

Standard	S-parameter
Unknown Reflection	$\mathbf{S} = \begin{pmatrix} r \end{pmatrix}$
Unknown Reflection 2	$\mathbf{S} = \begin{pmatrix} me^{-j4\pi fl/c} \end{pmatrix}$ with $\lvert m \rvert \leq 1$ and $l \geq 0$
Unknown Thru	$\mathbf{S} = \begin{pmatrix} r_1 & t \\ t & r2 \end{pmatrix}$
Unknown Isolation	$\mathbf{S} = \begin{pmatrix} r_1 & 0 \\ 0 & r2 \end{pmatrix}$
Unknown Line	$\mathbf{S} = \begin{pmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{pmatrix}$
Unknown Line 2	$\mathbf{S} = \begin{pmatrix} r_1 & e^{-\gamma l} \\ e^{-\gamma l} & r_2 \end{pmatrix}$ with $\gamma l = g_1 \sqrt{\frac{f}{10^9}} + g_2 \frac{f}{10^9}$
Unknown Series Inductance	$\mathbf{S} = \begin{pmatrix} \frac{R+j\omega L-Z_r}{R+j\omega L+Z_r} \end{pmatrix}$ with $L = ?$
Unknown Capacitance	$\mathbf{S} = \begin{pmatrix} \frac{1-j\omega CZ_r}{1+j\omega CZ_r} \end{pmatrix}$ with $C = ?$ and $C \geq 0$

6.6 Primary Airline Standard

The S-parameters of a primary airline standard are defined by the following equation

$$\mathbf{S} = \mathbf{O} \oplus \mathbf{K_1} \oplus \mathbf{C_1} \oplus \mathbf{L_1} \oplus \mathbf{P} \oplus \mathbf{L_2} \oplus \mathbf{C_2} \oplus \mathbf{K_2} \oplus \mathbf{O}^{\ominus 1}. \tag{193}$$

$\mathbf{O}$ denotes the line shift, see section 6.6.1.

$\mathbf{K_1}$ denotes the kapton or adapter effect on port 1.

$\mathbf{C_1}$ denotes the half connector of the standard at port 1.

$\mathbf{L_1}$ denotes the half of the line section on port 1 side.

$\mathbf{P}$ denotes a section of ideal line, see section 6.6.2.

$\mathbf{L_2}$ denotes the half of the line section on port 2 side.

$\mathbf{C_2}$ denotes the half connector of the standard at port 2.

$\mathbf{K_2}$ denotes the kapton or adapter effect on port 2.

6.6.1 Line Shift

The S-parameters of the line shift are defined by the following equation

$$\mathbf{O} = \begin{pmatrix} 0 & e^{-j\frac{2\pi f}{c_0} l_{shift}} \\ e^{-j\frac{2\pi f}{c_0} l_{shift}} & 0 \end{pmatrix} \tag{194}$$

where f is the frequency, c_0 is the speed of light and l_{shift} is the shift length. The line shift section has to be used when the center conductor protrudes in a test port.

6.6.2 Propagation Constant

The following equation describes the propagation constant

$$\gamma l = g_1 \sqrt{\frac{f}{1\,\mathrm{GHz}}} + g_2 \frac{f}{1\,\mathrm{GHz}}. \tag{195}$$

g_1, g_2 are the unknown parameters and f is the frequency. The resulting line section is defined by the following equation

$$\mathbf{P} = \begin{pmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{pmatrix}. \tag{196}$$

6.7 Primary Offset Short Standard

The S-parameters of a primary offset short standard are defined by the following equation

$$\mathbf{S} = \mathbf{K_1} \oplus \mathbf{C_1} \oplus \mathbf{L_1} \oplus \mathbf{P} \oplus \mathbf{L_2} \oplus \mathbf{T}. \tag{197}$$

$\mathbf{K_1}$ denotes the kapton or adapter effect on port 1.

$\mathbf{C_1}$ denotes the half connector of the standard.

$\mathbf{L_1}$ denotes the half of the line section on the connector side.

$\mathbf{P}$ denotes a section of ideal line, see section 6.6.2.

$\mathbf{L_2}$ denotes the half of the line section on the side of the short plane.

$\mathbf{T}$ denotes the short plane, see section 6.7.1.

6.7.1 Short Plane

The short plane is defined by the following equations

$$Z = z_1 + z_2\sqrt{\frac{f}{1\,\mathrm{GHz}}} + z_3\frac{f}{1\,\mathrm{GHz}} \tag{198}$$

$$T_{11} = \frac{Z - Z_r}{Z + Z_r} \tag{199}$$

where z_1, z_2, z_3 are the unknown parameters, f is the frequency and Z_r is the reference impedance.

6.8 Primary Flush Short Standard

The S-parameters of a primary flush short standard are defined by the following equation

$$\mathbf{S} = \mathbf{C_1} \oplus \mathbf{T}. \tag{200}$$

$\mathbf{C_1}$ denotes the half connector of the standard.

$\mathbf{T}$ denotes the short plane, see section 6.7.1.

6.9 Waveguide Shim Standard

The S-parameters of a waveguide shim standard are defined by the following equation

$$\mathbf{S} = \mathbf{O_1} \oplus \mathbf{C_1} \oplus \mathbf{D} \oplus \mathbf{C_2} \oplus \mathbf{O_2}. \tag{201}$$

$\mathbf{O_1}$ denotes the effect related to the offset (vertical and horizontal) of the waveguide connector at port 1, see section 6.9.2.

$\mathbf{C_1}$ denotes the half connector of the standard at port 1, see section 6.9.1.

$\mathbf{D}$ denotes the shim, see section 6.9.3.

$\mathbf{C_2}$ denotes the half connector of the standard at port 2, see section 6.9.1.

$\mathbf{O_2}$ denotes the effect related to the offset (vertical and horizontal) of the waveguide connector at port 2, see section 6.9.2.

6.9.1 Waveguide Connector

The S-parameters of the waveguide connector are computed using a transmission line junction, see appendix A.2.1. The impedances of the test port Z_1 and of the calibration standard Z_2 are defined by the following equations

$$Z_1 = j\frac{2\pi f \mu_1}{\gamma_1} \tag{202}$$

$$Z_2 = j\frac{2\pi f \mu_2}{\gamma_2} \tag{203}$$

where f is the frequency, μ the permeability and γ the propagation constant of the waveguide section, see section 6.9.4.

6.9.2 Waveguide Connector Offset

The S-parameters of the offset (width and height) of the waveguide connector are based on a look up database which has been computed using COMSOL. The following limitations exist:

- The offset in direction of the width has to be between 0 % and 3.1496 % (corresponds to $80\,\mu m$ in WR10) of the nominal width.
- The offset in direction of the height has to be between 0 % and 6.2992 % (corresponds to $80\,\mu m$ in WR10) of the nominal height.
- The width height ratio has to be between 2 and 2.5.

6.9.3 Shim

The shim section is defined by the following equation

$$\mathbf{D} = \begin{pmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{pmatrix}. \tag{204}$$

where l is the length of the shim section and γ is the propagation constant, see section 6.9.4. The unknown parameters in an optimization calibration are the length l and the conductivity σ_{DC}, which is used to compute the propagation constant γ.

6.9.4 Gamma

The propagation constant of a rectangular waveguide is described in [20]. The following equations are needed:

$$\sigma = \sigma_{DC} - \sigma_{HF}\sqrt{\frac{f}{1\,\mathrm{GHz}}} \tag{205}$$

$$w_e = w - \frac{(4-\pi)\,r^2}{h} \tag{206}$$

$$x_0 = \left(\frac{c_0}{2w_e f}\right)^2 \tag{207}$$

$$x_1 = \frac{2\pi f\sqrt{\epsilon_r}}{c_0} \tag{208}$$

$$x_2 = \sqrt{1-x_0} \tag{209}$$

$$x_3 = \frac{\sqrt{\frac{\pi f \mu_0}{\sigma}}}{h} \tag{210}$$

$$x_4 = \sqrt{\frac{\epsilon_0}{\mu_0}} \tag{211}$$

$$x_5 = 1 + \frac{2h}{w_e}x_0 \tag{212}$$

$$\gamma = \frac{x_3 x_4 x_5}{x_2} + j x_1 x_2. \tag{213}$$

μ and ϵ are permeability and permittivity. The waveguide section is characterized by his conductivity σ_{DC} and σ_{HF}. The frequency is f. Width w, height h and radius r describe the geometry of the waveguide section.

6.10 Waveguide Offset Short Standard

The S-parameters of a waveguide offset short standard are defined by the following equation

$$\mathbf{S} = \mathbf{O_1} \oplus \mathbf{C_1} \oplus \mathbf{D} \oplus \mathbf{T}. \tag{214}$$

$\mathbf{O_1}$ denotes the effect related to the offset (vertical and horizontal) of the waveguide connector at port 1, see section 6.9.2.

$\mathbf{C_1}$ denotes the half connector of the standard, see section 6.9.1.

$\mathbf{D}$ denotes the shim, see section 6.9.3.

$\mathbf{T}$ denotes the short plane, see section 6.10.1.

6.10.1 Short Plane

The short plane is defined by the following equations

$$Z = z_1 + z_2\sqrt{\frac{f}{1\,\mathrm{GHz}}} + z_3\frac{f}{1\,\mathrm{GHz}} \tag{215}$$

$$T_{11} = \frac{Z - Z_r}{Z + Z_r} \tag{216}$$

where z_1, z_2, z_3 are the unknown parameters, f is the frequency and Z_r is the reference impedance.

6.11 Waveguide Flush Short Standard

The S-parameters of a waveguide flush short standard are defined by the following equation

$$\mathbf{S} = \mathbf{T}. \tag{217}$$

$\mathbf{T}$ denotes the short plane, see section 6.10.1.

6.12 Simple Line Standard

The distributed admittance is computed with the following equation

$$Y' = G' + j\omega C' \tag{218}$$

where G' and C' are specified by the user per frequency point. The transmission line section is computed with the following two equations

$$\gamma l = \left(x_1 \sqrt{\epsilon_r} + j x_2 \frac{\omega}{c_0} \sqrt{\epsilon_r} \right) l \tag{219}$$

$$Z_0 = \frac{\gamma}{Y'} \tag{220}$$

where l is the length of the line, c_0 is the speed of light, ϵ_r is the relative permittivity and x_1, x_2 are the unknown parameters for each frequency. For computing the S-parameters of a transmission line section see section A.2.2.

6.13 On Wafer Line Standard

The start values for the characteristic impedance Z_0' and the propagation constant γ' are computed using the Heinrich model which is described in [21]. The distributed admittance is computed with the following equation.

$$Y' = \frac{\gamma'}{Z_0'} \tag{221}$$

The transmission line section is computed with the following two equations

$$\gamma l = \left(x_1 + \mathrm{Re}\left(\gamma'\right) + j x_2 \,\mathrm{Im}\left(\gamma'\right) \right) l \tag{222}$$

$$Z_0 = \frac{\gamma}{Y'} \tag{223}$$

where l is the length of the line and x_1, x_2 are the unknown parameters for each frequency. For computing the S-parameters of a transmission line section see section A.2.2.

6.14 On Wafer Offset Short Standard

The S-parameters of an on wafer offset short standard are defined by the following equation

$$\mathbf{S} = \mathbf{L} \oplus \mathbf{T}. \tag{224}$$

$\mathbf{L}$ denotes the offset line section, see section 6.14.1.

$\mathbf{T}$ denotes the short plane, see section 6.14.2.

6.14.1 Offset Line Section

The characteristic impedance Z_0 and the propagation constant γ of the offset line section are computed using the Heinrich model which is described in [21]. The length l is the unknown parameter. For computing the S-parameters of a transmission line section see section A.2.2.

6.14.2 Short Plane

The short plane is defined by the following equations

$$Z = z_1 + z_2\sqrt{\frac{f}{1\,\mathrm{GHz}}} + z_3\frac{f}{1\,\mathrm{GHz}} \tag{225}$$

$$T_{11} = \frac{Z - Z_r}{Z + Z_r} \tag{226}$$

where z_1, z_2, z_3 are the unknown parameters, f is the frequency and Z_r is the reference impedance.

6.15 On Wafer Flush Short Standard

The S-parameters of an on wafer flush short standard are defined by the following equation

$$\mathbf{S} = \mathbf{T}. \tag{227}$$

$\mathbf{T}$ denotes the short plane, see section 6.14.2.

7 VNA Uncertainty Contributions

Tables 4, 5, 6, 7 and 8 show the uncertainty input ids.

7.1 Noise and linearity

The noise influence is uncorrelated for each measurement because it's a random effect. The linearity influence is correlated for each measurement because it's a systematic effect.

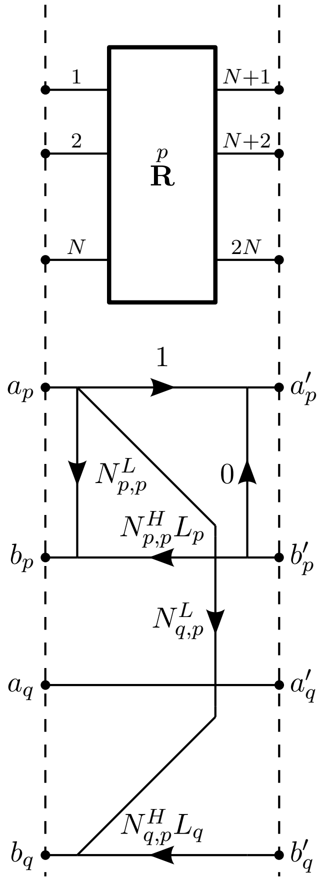

Figure 10: Noise and linearity influences

Uncertainty definition:

$N^L_{p,p}$ is the noise floor in dB (additive) of source port p.

$N^L_{q,p}$ is the noise floor in dB (additive) from port p to q.

$N^H_{p,p}$ is the trace noise in dB and deg (multiplicative) of source port p.

$N^H_{q,p}$ is the trace noise in dB and deg (multiplicative) from port p to q.

L_p, L_q are the linearity in dB and deg (multiplicative) of port p and port q.

7.2 Drift of switch and error terms

The structure of $\mathbf{D}$ can but must not be a copy of $\mathbf{E}$. The drift influence $\mathbf{D}$ is acting on corrected S-parameters. Those the specification of $\mathbf{D}$ should be for the drift of the corrected S-parameters.

The individual drift contributions for the switch and error terms are uncorrelated for each measurement. The single drift terms are partly correlated over time. E.g.: the drift in directivity of a measurement i and $i+1$ are partly correlated whereas there is no correlation between directivity and tracking drift.

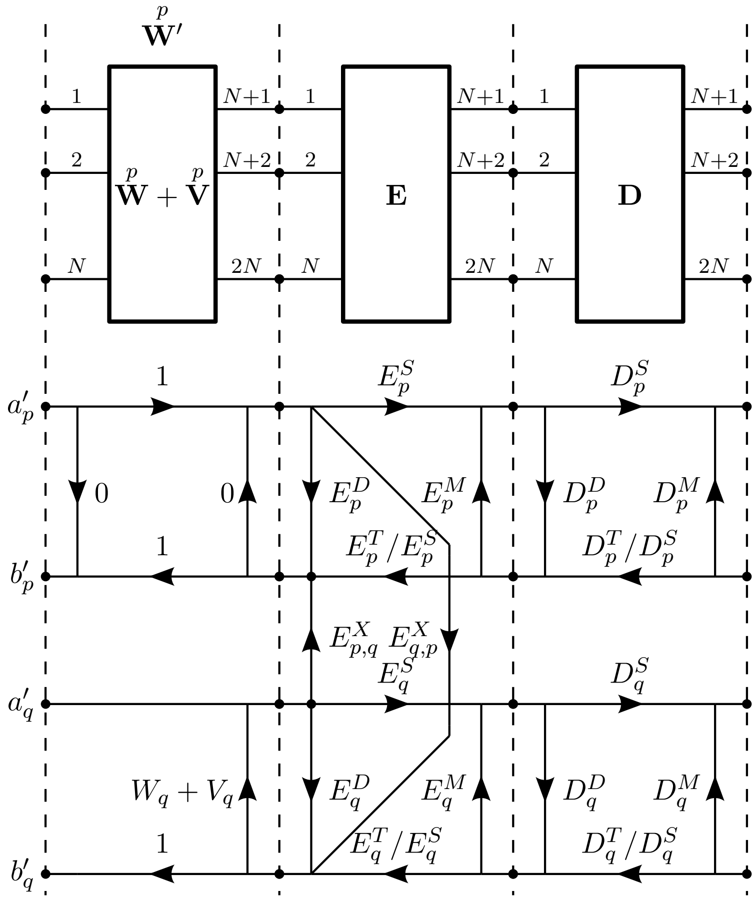

Figure 11: Drift of switch and error terms

Uncertainty definition:

V_q is the switch term drift in dB (additive) of port q when p is the source port.

D_p^D is the directivity drift in dB (additive) of port p.

D_p^T is the tracking drift in dB and deg (multiplicative) of port p.

D_p^S is the symmetry drift in dB and deg (multiplicative) of port p.

D_p^M is the match drift in dB (additive) of port p.

7.3 Cable stability, connector repeatability and DUT uncertainty

The cable influences are uncorrelated for each new cable position. The connector influences are uncorrelated for each new connection. The DUT influences are uncorrelated for each new DUT index.

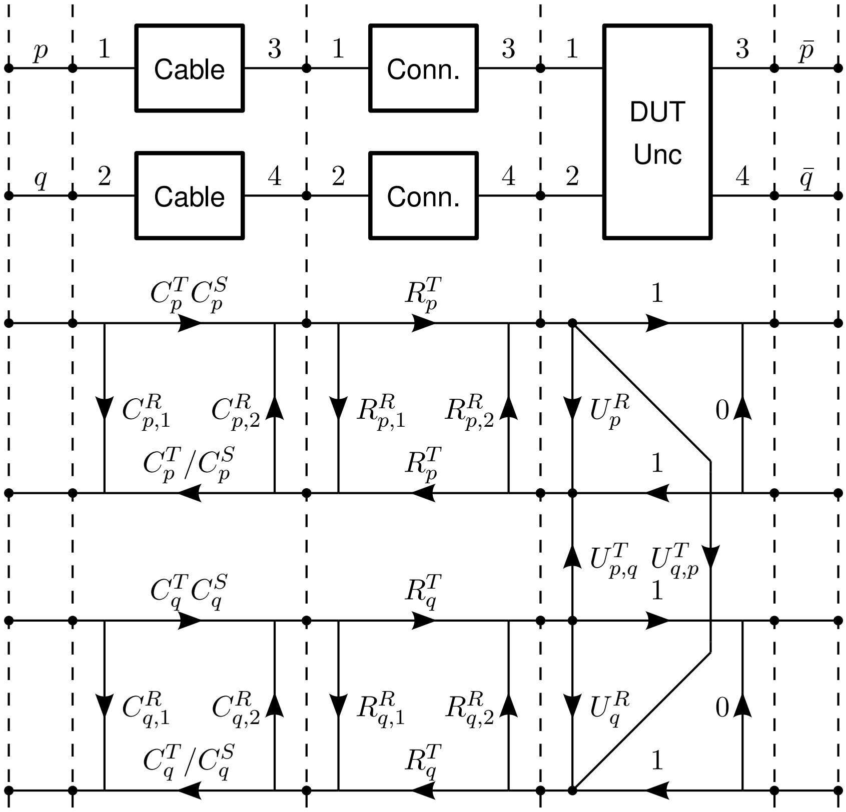

Figure 12: Cable stability, connector repeatability and DUT uncertainty

Uncertainty definition:

$C_{p,1}^R, C_{p,2}^R$ is the cable reflection stability in dB (additive) of port p.

C_p^T is the cable transmission stability in dB and deg (multiplicative) of port p.

C_p^S is the cable symmetry stability in dB and deg (multiplicative) of port p.

$R_{p,1}^R, R_{p,2}^R$ is the connector reflection repeatability in dB (additive) of port p.

R_p^T is the connector transmission repeatability in dB and deg (multiplicative).

U_p^R is the DUT reflection uncertainty (additive) of port p.

$U_{p,q}^T$ is the DUT transmission uncertainty (additive) from port q to p.

The DUT uncertainty can be used to represent the crosstalk in on-wafer measurements. In this case the DUT uncertainty is uncorrelated for every standard on the wafer and it's correlated for multiple measurements of the same standard.

Table 4: Uncertainty Input IDs

Unc Contribution	Global ID 128 bit	Influence 16 bit	Reserve 8 bit	Version 8 bit	Counter 63–48 bit	47–40 bit	39–32 bit	31–1 bit	0 bit
Unknown	Random ID	0x0000	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
CMC Entry	CMC ID	0x0001	0x00	0x00	Random ID	Rcv Port	Src Port	Freq	RI
CMC Entry	CMC ID	0x0002	0x00	0x00	Random ID	Rcv Port	Src Port	Freq	MP
VNA Exp Statistical	Journal ID	0x0008	0x00	0x00	Meas Count	Rcv Port	Src Port	Freq	RI
	Journal ID	0x0008	0x00	0x01, 0x02	Meas Count	Contribution		Freq	0
VNA Exp Systematic	Journal ID	0x0009	0x00	0x00	Exp Count	Rcv Port	Src Port	Freq	RI
	Journal ID	0x0009	0x00	0x01, 0x02	Exp Count	Contribution		Freq	0
VNA Noise Floor	Journal ID	0x0011	0x00	0x00	Meas Count	Rcv Port	Src Port	Freq	RI
VNA Noise Floor	Journal ID	0x0011	0x00	0x01	Meas Count	Swt Port	0	Freq	RI
VNA Noise Trace	Journal ID	0x0012	0x00	0x00	Meas Count	Rcv Port	Src Port	Freq	MP
VNA Noise Trace	Journal ID	0x0012	0x00	0x01	Meas Count	Swt Port	0	Freq	MP
VNA Linearity	VNA ID	0x0020	0x00	0x00	$100(p+320)$	Rcv Port	0x00	0x00000000	MP
	VNA ID	0x0020	0x00	0x01, 0x02	0x0000	Rcv Port	0x00	$10^6(p+1000)$	MP
	VNA ID	0x0020	0x00	0x03	0x0000	Swt Port	0x00	$10^6(p+1000)$	MP
VNA Drift Switch Term	Journal ID	0x0031	0x00	0x00, 0x01, 0x02, 0x03	Meas Count	Port	0x00	Freq	RI
VNA Drift Directivity	Journal ID	0x0032	0x00	0x00, 0x01, 0x02, 0x03	Meas Count	Port	0x00	Freq	RI
VNA Drift Tracking	Journal ID	0x0033	0x00	0x00, 0x01, 0x02, 0x03	Meas Count	Port	0x00	Freq	MP
VNA Drift Match	Journal ID	0x0034	0x00	0x00, 0x01, 0x02, 0x03	Meas Count	Port	0x00	Freq	RI
VNA Drift Isolation	Journal ID	0x0035	0x00	0x00, 0x01, 0x02, 0x03	Meas Count	Rcv Port	Src Port	Freq	RI
VNA Drift Symmetry	Journal ID	0x0036	0x00	0x02, 0x03	Meas Count	Port	0x00	Freq	MP

Table 5: Uncertainty Input IDs cont.

Unc Contribution	Global ID 128 bit	Influence 16 bit	Reserve 8 bit	Version 8 bit	Counter 63–48 bit	47–40 bit	39–32 bit	31–1 bit	0 bit
Cable Transmission	Journal ID	0x0040	0x00	0x00	Cable Pos	Port	0x00	Freq	MP
Cable Reflection	Journal ID	0x0041	0x00	0x00	Cable Pos	Port	C_1^R, C_2^R	Freq	RI
Cable Symmetry	Journal ID	0x0042	0x00	0x00	Cable Pos	Port	0x00	Freq	MP
Connector Reflection	Journal ID	0x0050	0x00	0x00	Conn Count	Port	R_1, R_2	Freq	RI
Connector Transmission	Journal ID	0x0051	0x00	0x00	Conn Count	Port	0x00	Freq	MP
DUT Uncertainty	Journal ID	0x0058	0x00	0x00	DUT Index	Rcv Port	Src Port	Freq	RI
Cal Std Short	Cal Std ID	0x0061	0x00	0x00	0x0000	0x00	0x00	0x00000000	MP
Cal Std Open	Cal Std ID	0x0062	0x00	0x00	0x0000	0x00	0x00	0x00000000	MP
Cal Std Load	Cal Std ID	0x0063	0x00	0x00	0x0000	0x00	0x00	0x00000000	RI
Cal Std Thru / Delay Refl.	Cal Std ID	0x0064	0x00	0x00	0x0000	0x00	0x00	0x00000000	RI
Cal Std Short	Cal Std ID	0x0061	0x00	0x01	Frequency (mHz)				MP
Cal Std Open	Cal Std ID	0x0062	0x00	0x01	Frequency (mHz)				MP
Cal Std Load	Cal Std ID	0x0063	0x00	0x01	Frequency (mHz)				RI
Cal Std Thru / Delay Refl.	Cal Std ID	0x0064	0x00	0x01	Frequency (mHz)				RI
Cal Std Thru / Delay Trans.	Cal Std ID	0x0065	0x00	0x01	Frequency (mHz)				MP
Agilent Unc Calculator	Random ID	0x0071	0x00	0x00	0x0000	Rcv Port	Src Port	Freq	RI
Agilent Unc CITI File RI	Data ID	0x0072	0x00	0x00	0x0000	Rcv Port	Src Port	Freq	RI
Agilent Unc CITI File MP	Data ID	0x0073	0x00	0x00	0x0000	Rcv Port	Src Port	Freq	MP
Electrical Resistance	Random ID	0x0093	0x00	0x00	0x0000	0x00	0x00	Contribution	
TD Unknown DC Point	Random ID	0x0100	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
TD Unknown Frequency Point	Data ID	0x0101	0x00	0x00	Frequency (mHz)				RI
Data Set	Data ID	0x0200	0x00	0x00	Freq			Contribution	

Table 6: Uncertainty Input IDs cont.

Unc Contribution	Global ID	Influence	Reserve	Version	Counter				
	128 bit	16 bit	8 bit	8 bit	63–48 bit	47–40 bit	39–32 bit	31–1 bit	0 bit
Material Relative Permittivity	Cal Std ID	0x0400	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Material Relative Permeability	Cal Std ID	0x0401	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Material Conductivity	Cal Std ID	0x0402	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Material DC Conductivity	Cal Std ID	0x0403	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Material HF Conductivity	Cal Std ID	0x0404	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Material Tan Delta	Cal Std ID	0x0405	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Conn Pin Depth	Cal Std ID	0x0410	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Pin Gap	Cal Std ID	0x0411	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Outer Chamfer	Cal Std ID	0x0412	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Inner Chamfer	Cal Std ID	0x0413	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Male Outer Chamfer	Cal Std ID	0x0414	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Male Inner Chamfer	Cal Std ID	0x0415	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Male Pin Diameter	Cal Std ID	0x0416	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Hole Diameter	Cal Std ID	0x0417	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Outer Conductor Diameter	Cal Std ID	0x0418	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Male Center Conductor Diameter	Cal Std ID	0x0419	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Hole Length	Cal Std ID	0x0420	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Center Conductor Diameter	Cal Std ID	0x0421	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Slot Length	Cal Std ID	0x0430	0x00	0x00	Port	0x00	0x00	0x00000000	0
Conn Female Slot Width	Cal Std ID	0x0431	0x00	0x00	Port	0x00	0x00	0x00000000	0

Table 7: Uncertainty Input IDs cont.

Unc Contribution	Global ID 128 bit	Influence 16 bit	Reserve 8 bit	Version 8 bit	Counter 63–48 bit	47–40 bit	39–32 bit	31–1 bit	0 bit
Conn Uncompressed Mid Finger Diameter	Cal Std ID	0x0441	0x00	0x00	Port	0x00	0x00	Pos Index	0
Conn Compresseed Mid Finger Diameter	Cal Std ID	0x0442	0x00	0x00	Port	0x00	0x00	Pos Index	0
Conn Outer Diameterin Finger Sections	Cal Std ID	0x0443	0x00	0x00	Port	0x00	0x00	Pos Index	0
Connector FDTD Real	Cal Std ID	0x0450	0x00	0x00	Port	0x00	0x00	0x00000000	0
Connector FDTD Imag	Cal Std ID	0x0451	0x00	0x00	Port	0x00	0x00	0x00000000	0
Pin Gap FDTD Real	Cal Std ID	0x0452	0x00	0x00	Port	0x00	0x00	0x00000000	0
Pin Gap FDTD Imag	Cal Std ID	0x0453	0x00	0x00	Port	0x00	0x00	0x00000000	0
Standard Length	Cal Std ID	0x0460	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line Length	Cal Std ID	0x0461	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line z Position	Cal Std ID	0x0462	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line ICOD	Cal Std ID	0x0463	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line OCID	Cal Std ID	0x0464	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line Attenuation Constant	Cal Std ID	0x0470	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Line Phase Constant	Cal Std ID	0x0471	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Short Plane ICOD	Cal Std ID	0x0480	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Short Plane OCID	Cal Std ID	0x0481	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Misc Line Shift	Cal Std ID	0x0490	0x00	0x00	0x0000	0x00	0x00	0x00000000	0

Table 8: Uncertainty Input IDs cont.

Unc Contribution	Global ID	Influence	Reserve	Version	Counter				
	128 bit	16 bit	8 bit	8 bit	63–48 bit	47–40 bit	39–32 bit	31–1 bit	0 bit
Waveguide Length	Cal Std ID	0x0500	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Width	Cal Std ID	0x0501	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Height	Cal Std ID	0x0502	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Radius	Cal Std ID	0x0503	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Width Offset	Cal Std ID	0x0504	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Height Offset	Cal Std ID	0x0505	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Waveguide Connector FEM	Cal Std ID	0x0506	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Simple Line Length	Cal Std ID	0x0600	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Simple Line G	Cal Std ID	0x0601	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
Simple Line C	Cal Std ID	0x0602	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
On Wafer Length	Cal Std ID	0x0610	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
On Wafer Width of Ground Conductor	Cal Std ID	0x0611	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
On Wafer Width of Signal Conductor	Cal Std ID	0x0612	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
On Wafer Gap Width	Cal Std ID	0x0613	0x00	0x00	0x0000	0x00	0x00	0x00000000	0
On Wafer Conductor Thickness	Cal Std ID	0x0614	0x00	0x00	0x0000	0x00	0x00	0x00000000	0

A S-Parameter Tools

A.1 Cascading

Cascading of two S-parameter sets is described in [22].

A.1.1 Cascading of a 2N-Port and a N-Port

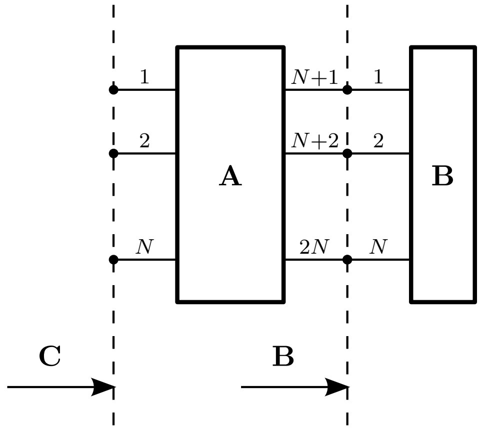

Figure 13: Cascading of a $2N$-port ($\mathbf{A}$) and a N-port ($\mathbf{B}$)

One can use the following equation to cascade a $2N$-port and a N-port. The result will be the N-port $\mathbf{C}$

$$\mathbf{C} = \mathbf{A}_{00} + \mathbf{A}_{01}\left(\mathbf{I} - \mathbf{B}\mathbf{A}_{11}\right)^{-1}\mathbf{B}\mathbf{A}_{10} \tag{228}$$

with

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{00} & \mathbf{A}_{01} \\ \mathbf{A}_{10} & \mathbf{A}_{11} \end{pmatrix}. \tag{229}$$

The variables $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are S-parameter matrices. $\mathbf{A}$ is a $2N$-port and $\mathbf{B}$ is a N-port. A simplified notation can be achieved by introducing a new operator

$$\mathbf{C} = \mathbf{A} \oplus \mathbf{B}. \tag{230}$$

To find the reverse function, equation 228 can be rewritten as

$$\mathbf{A}_{01}^{-1}\left(\mathbf{C} - \mathbf{A}_{00}\right)\mathbf{A}_{10}^{-1} = \left(\mathbf{I} - \mathbf{B}\mathbf{A}_{11}\right)^{-1}\mathbf{B}. \tag{231}$$

Setting

$$\mathbf{X} = \mathbf{A}_{01}^{-1}\left(\mathbf{C} - \mathbf{A}_{00}\right)\mathbf{A}_{10}^{-1} \tag{232}$$

and rearranging equation (231) yields

$$\mathbf{B} = \mathbf{X}\left(\mathbf{I} + \mathbf{A}_{11}\mathbf{X}\right)^{-1}. \tag{233}$$

Equations (232) and (233) can be used to decascade the $2N$-port $\mathbf{A}$ from N-port $\mathbf{C}$ and to obtain the N-port $\mathbf{B}$. Similar to the notation in equation (230) a new operator can be introduced

$$\mathbf{B} = \mathbf{C} \ominus \mathbf{A}. \tag{234}$$

A.1.2 Cascading of a 2-Port and a N-Port

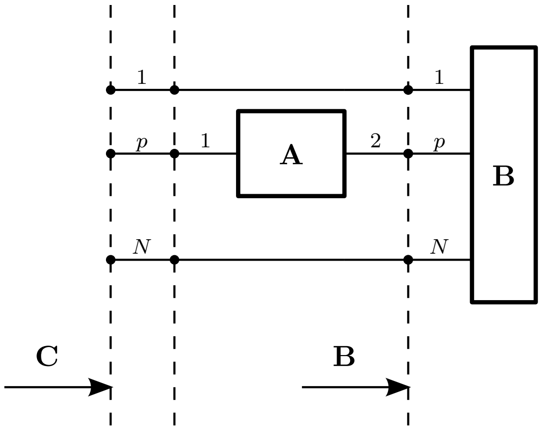

Figure 14: Cascading a of 2-port ($\mathbf{A}$) and a N-port ($\mathbf{B}$)

Cascading of a 2-port and a N-port can be computed with

$$C_{ij} = \begin{cases} A_{11} + \frac{A_{21} B_{ij} A_{12}}{1 - A_{22} B_{pp}} & , i = j = p \\ \frac{A_{21} B_{ij}}{1 - A_{22} B_{pp}} & , i \neq p \wedge j = p \\ \frac{B_{ij} A_{12}}{1 - A_{22} B_{pp}} & , i = p \wedge j \neq p \\ B_{ij} + \frac{B_{pj} A_{22} B_{ip}}{1 - A_{22} B_{pp}} & , i \neq p \wedge j \neq p \wedge i \neq j \end{cases} . \tag{235}$$

Here the variables $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ denote S-parameter matrices. The result $\mathbf{C}$ is a N-port with a 2-port cascaded to port p of the original N-port $\mathbf{B}$.

A.1.3 Cascading of a 2-Port and a 2-Port

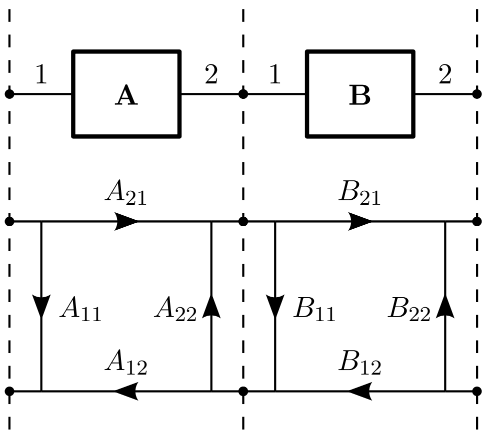

Figure 15: Cascading of a 2-port ($\mathbf{A}$) and a 2-port ($\mathbf{B}$)

A special case of cascading is a 2-port and a 2-port. Equation (283) can be rewritten as

$$C_{11} = A_{11} + \frac{A_{21}B_{11}A_{12}}{1 - A_{22}B_{11}} \tag{236}$$

$$C_{21} = \frac{A_{21}B_{21}}{1 - A_{22}B_{11}} \tag{237}$$

$$C_{12} = \frac{B_{12}A_{12}}{1 - A_{22}B_{11}} \tag{238}$$

$$C_{22} = B_{22} + \frac{B_{12}A_{22}B_{21}}{1 - A_{22}B_{11}} \tag{239}$$

and a new operator can be introduced

$$\mathbf{C} = \mathbf{A} \otimes \mathbf{B}. \tag{240}$$

To invert a 2-port, equations (236) to (239) can be rewritten as

$$B_{11} = \frac{A_{11}}{A_{11}A_{22} - A_{21}A_{12}} \tag{241}$$

$$B_{21} = \frac{1 - A_{22}B_{11}}{A_{21}} \tag{242}$$

$$B_{12} = \frac{1 - A_{22}B_{11}}{A_{12}} \tag{243}$$

$$B_{22} = -\frac{B_{12}A_{22}B_{21}}{1 - A_{22}B_{11}} \tag{244}$$

with $C_{11} = C_{22} = 0$ and $C_{21} = C_{12} = 1$. And a new operator for inverting a 2-port can be introduced.

$$\mathbf{B} = \mathbf{A}^{\ominus 1}. \tag{245}$$

A.1.4 Cascading of a 2-Port and a 1-Port

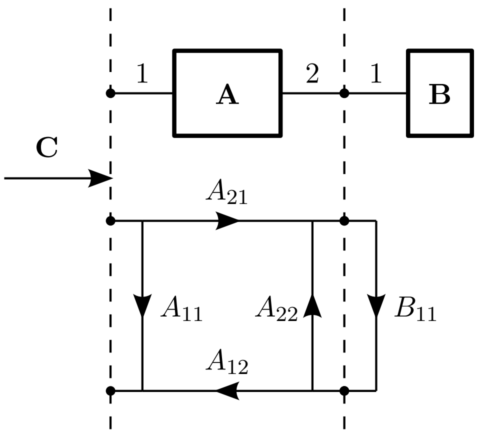

Figure 16: Cascading a 2-port ($\mathbf{A}$) and a 1-port ($\mathbf{B}$)

Another special case of cascading is a 2-port and a 1-port. Then equation (283) can be rewritten as

$$C_{11} = A_{11} + \frac{A_{21}B_{11}A_{12}}{1 - A_{22}B_{11}}. \tag{246}$$

A.2 Transmission Line

A.2.1 Transmission Line Junction

A transmission line junction 2-port can be used to change the reference impedance. The transformation of the reference impedance is described in [23].

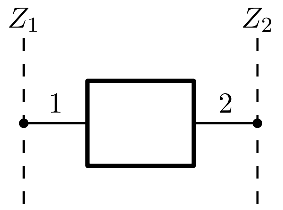

Figure 17: Transmission line junction

The S-parameters of a transmission line junction are given by the following equations

$$S_{11} = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} \tag{247}$$

$$S_{21} = \frac{2Z_2 \left|\frac{Z_1}{Z_2}\right| \sqrt{\frac{\mathrm{Re}(Z_2)}{\mathrm{Re}(Z_1)}}}{(Z_1 + Z_2)} \tag{248}$$

$$S_{12} = \frac{2Z_1 \left|\frac{Z_2}{Z_1}\right| \sqrt{\frac{\mathrm{Re}(Z_1)}{\mathrm{Re}(Z_2)}}}{(Z_1 + Z_2)} \tag{249}$$

$$S_{22} = \frac{(Z_1 - Z_2)}{(Z_1 + Z_2)}. \tag{250}$$

The reference impedances of these S-parameters are Z_1 for port one and Z_2 for port two.

A.2.2 Transmission Line Section

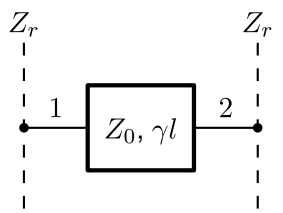

Figure 18: Transmission line section

The S-parameters of a transmission line section are given by the following equations

$$S_{11} = \frac{\left({Z_0}^2 - {Z_r}^2\right)\sinh\gamma l}{2Z_0 Z_r\cosh\gamma l + \left({Z_0}^2 + {Z_r}^2\right)\sinh\gamma l} \quad (251)$$

$$S_{21} = \frac{2Z_0 Z_r}{2Z_0 Z_r\cosh\gamma l + \left({Z_0}^2 + {Z_r}^2\right)\sinh\gamma l} \quad (252)$$

$$S_{12} = \frac{2Z_0 Z_r}{2Z_0 Z_r\cosh\gamma l + \left({Z_0}^2 + {Z_r}^2\right)\sinh\gamma l} \quad (253)$$

$$S_{22} = \frac{\left({Z_0}^2 - {Z_r}^2\right)\sinh\gamma l}{2Z_0 Z_r\cosh\gamma l + \left({Z_0}^2 + {Z_r}^2\right)\sinh\gamma l}. \quad (254)$$

Where Z_0 is the characteristic impedance and γl is the propagation constant times the length. The reference impedance at both ports is Z_r.

A.2.3 Lossy Coaxial Transmission Line Section

A lossy coaxial transmission line section is described in [24].

$$\sigma = \sigma_{DC} - \sigma_{HF}\sqrt{\frac{f}{1\,\mathrm{GHz}}} \quad (255)$$

$$k = \omega\sqrt{\mu\epsilon} \quad (256)$$

$$d_0 = \frac{\sqrt{\frac{2}{\sigma\omega\mu}}\left(1+\frac{b}{a}\right)}{4b\ln\left(\frac{b}{a}\right)} \quad (257)$$

$$F_0 = \frac{\frac{b^2}{a^2}-1}{2\ln\left(\frac{b}{a}\right)} - \frac{\frac{b}{a}\ln\left(\frac{b}{a}\right)}{\frac{b}{a}+1} - \frac{1}{2}\left(\frac{b}{a}+1\right) \quad (258)$$

$$C_0' = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \quad (259)$$

$$L_0' = \frac{\mu\ln\left(\frac{b}{a}\right)}{2\pi} \quad (260)$$

$$R' = 2\omega L_0' d_0\left(1 - \frac{k^2a^2F_0}{2}\right) \quad (261)$$

$$L' = L_0'\left(1 + 2d_0\left(1 - \frac{k^2a^2F_0}{2}\right)\right) \quad (262)$$

$$G' = \omega C_0' d_0 k^2 a^2 F_0 \quad (263)$$

$$C' = C_0'\left(1 + d_0 k^2 a^2 F_0\right) \quad (264)$$

$$Z' = R' + j\omega L' \quad (265)$$

$$Y' = G' + j\omega C' \quad (266)$$

$$\gamma = \sqrt{Z'Y'} \quad (267)$$

$$Z_0 = \sqrt{\frac{Z'}{Y'}} \quad (268)$$

Where μ and ϵ are permeability and permittivity of the dielectric. The conductors are characterized by their conductivity σ_{DC} and σ_{HF}. The frequency is f and the angular frequency is

ω. Outer conductor inner radius b and inner conductor outer radius a describe the geometry of the line.

B Time Domain

The here used transformation of frequency domain S-parameters to time domain and time gating is described in [25].

B.1 Frequency Domain to Time Domain

The transformation from frequency to time domain is shown in figure 19.

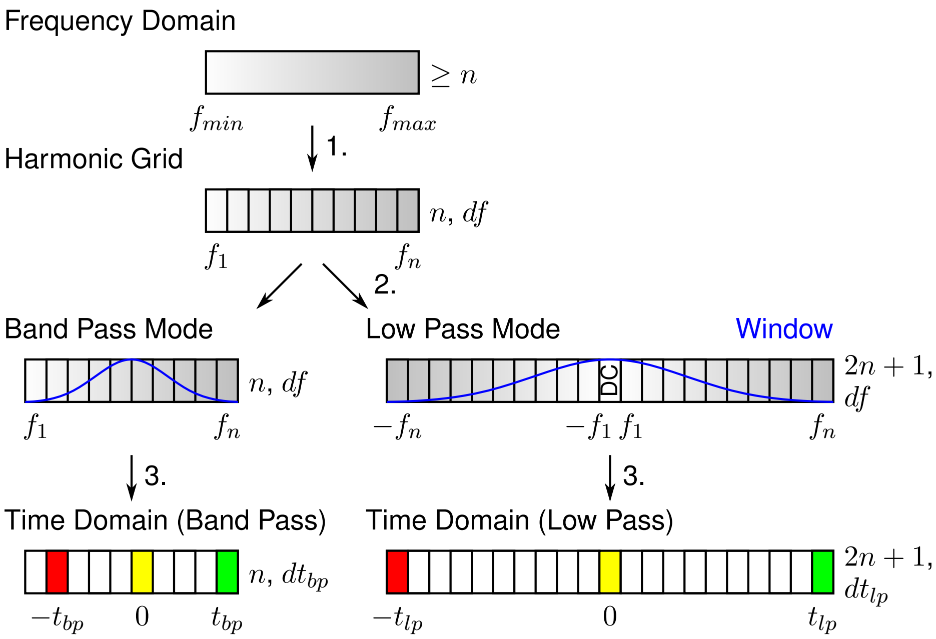

Figure 19: Illustration of the transformation from frequency domain to time domain using the band pass mode or the low pass mode.

1. The original data in frequency domain is interpolated on a harmonic grid. A harmonic grid is formed by a set of equidistant frequency points f_i $(i = 1 \ldots n)$ with spacing df. The frequency step df of the harmonic grid is equal to the largest frequency step of the original data in frequency domain. The first frequency f_1 of the harmonic grid has to be a multiple of df and $f_{min} \leq f_1$. The last frequency $f_n = f_{max}$ of the harmonic grid has to be a multiple of df.

2. In the low pass mode, the data of the harmonic grid is mirrored (conjugate complex) to the negative frequencies and the DC point is added.

3. The data on the harmonic grid is multiplied with the window function and then transformed to time domain using the inverse DFT (discrete fourier transform).

B.1.1 Band Pass Mode

The band pass mode simulates a narrow-band TDR (Time Domain Reflectometer). It allows the user to identify locations of mismatches but does not indicate whether the mismatches are capacitive, inductive or resistive. However, it's suitable for displaying a magnitude proportional to the response of a TDR. Since the band pass mode does not include a DC value and low frequency values, only the impulse excitation is supported.

The time resolution is computed with the following equation.

$$dt_{bp} = \frac{1}{n \cdot df} \tag{269}$$

Note that the time resolution gets finer with a larger frequency span $f_n - f_1$.

The maximum time is computed with the following equation.

$$t_{bp} = \left\lfloor \frac{n-1}{2} \right\rfloor \cdot dt_{bp} \tag{270}$$

The operation $\lfloor\ \rfloor$ denotes rounding to the lower integer number. Note that the maximum time gets larger with a smaller frequency step df.

The minimum time is computed with the following equation.

$$-t'_{bp} = -\left\lfloor \frac{n}{2} \right\rfloor \cdot dt_{bp} \tag{271}$$

Note that for an odd number of points n, it is $t'_{bp} = t_{bp}$.

B.1.2 Low Pass Mode

The low pass mode is used to simulate a traditional TDR measurement. This mode gives the user information to determine the type of discontinuity (R, L, or C) that is present. Low pass mode provides the best resolution (fastest rise time), and it my be used to either compute the step or impulse response of a device.

The low pass mode is less general than the band pass mode in that it places strict limitations on the frequency range of the measurement. The first frequency f_1 must be equal to the frequency step df. The DC frequency response is either measured or extrapolated from the two lowest frequency points in frequency domain. The requirement for a DC point is the same limitation that exists for traditional TDR measurements.

The time resolution of the low pass mode is computed with the following equation.

$$dt_{lp} = \frac{1}{(2n+1) \cdot df} \tag{272}$$

Note that the time resolution of the low pass mode is about twice as fine as the time resolution of the band pass mode.

The maximum and minimum time is computed with the following equation.

$$t_{lp} = n \cdot dt_{lp} \tag{273}$$

B.1.3 Frequency and Time Uncertainty

In the following example the band pass mode is used. Let's assume a frequency uncertainty of

$$\mathrm{u}\left(f_{stab}\right) = 10^{-6}\,\frac{Hz}{Hz} \tag{274}$$

and a frequency step $df = 100\,MHz$ and $n = 500$ measurement points. The time resolution would be computed with

$$dt_{bp} = \frac{1}{n \cdot df} = 20\,ps \pm 20\,as. \tag{275}$$

The maximum time would be

$$t_{bp} = \left\lfloor \frac{n-1}{2} \right\rfloor \cdot dt_{bp} = 4.98\,ns \pm 4.98\,fs \tag{276}$$

and the minimum time would be

$$-t'_{bp} = -\left\lfloor \frac{n}{2} \right\rfloor \cdot dt_{bp} = -5.00\,ns \pm 5.00\,fs. \tag{277}$$

The ratio of the worst case time uncertainty to the time resolution is

$$\frac{\mathrm{u}\left(t_{bp} + t'_{bp}\right)}{dt_{bp}} = \frac{9.98\,fs}{20\,ps} = (n-1)\,\mathrm{u}\left(f_{stab}\right) = (500-1)\,10^{-6}\,\frac{Hz}{Hz} = 4.99 \cdot 10^{-4}. \tag{278}$$

Note that the maximum and minimum time get larger the more measurement points n are used. Therefore the above ratio gets as well larger (worse) when more points n are used. The uncertainty of frequency and time is not taken into account by VNA Tools.

B.2 Time Gating

This section describes the transformation from frequency to time domain, gating the data in time domain and transforming back to frequency domain.

B.2.1 Band Pass Mode

For time gating, using the band pass mode, see figure 20.

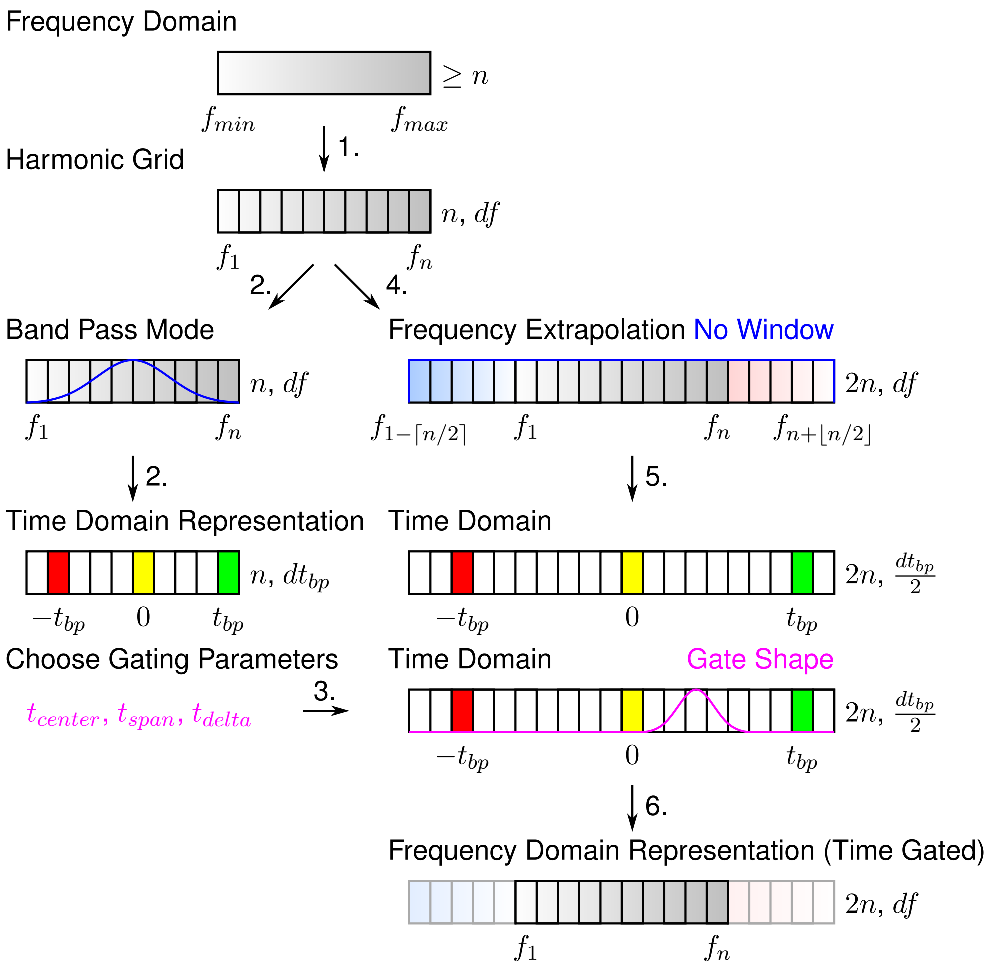

Figure 20: Illustration of time gating using the band pass mode. The steps 2 and 3 on the left side are used for time domain representation and to choose the gating parameters of the gate shape. The steps 4, 5 and 6 on the right side are used for the time gating process.

1. The original data in frequency domain is interpolated on a harmonic grid, see as well section B.1.

2. The data on the harmonic grid is multiplied with the window function and then transformed to time domain using the inverse DFT (discrete fourier transform).

3. The gating parameters t_{center}, t_{span} and t_{delta} are chosen. See section B.2.3 for computing the gate shape.

4. The data on the harmonic grid is extrapolated at the lower and higher frequency ranges. The extrapolated points are a mirrored version of the harmonic grid and have large standard uncertainties of $\pm(1+|v_{re}|)/\sqrt{3}$ in real part and $\pm(1+|v_{im}|)/\sqrt{3}$ in imaginary part where $|v_p|$ is the absolute value of the part p. They are used to regularize the value and compute the uncertainty at the boundaries of the time gated data in frequency domain.

5. The extrapolated data in frequency domain is transformed to time domain without applying any window function.

6. The complex data in time domain from step 5 is multiplied with the gate shape and then transformed back to frequency domain. A subset of the frequencies is the result (time gated data in frequency domain).

B.2.2 Low Pass Mode

For time gating, using the low pass mode, see figure 21.

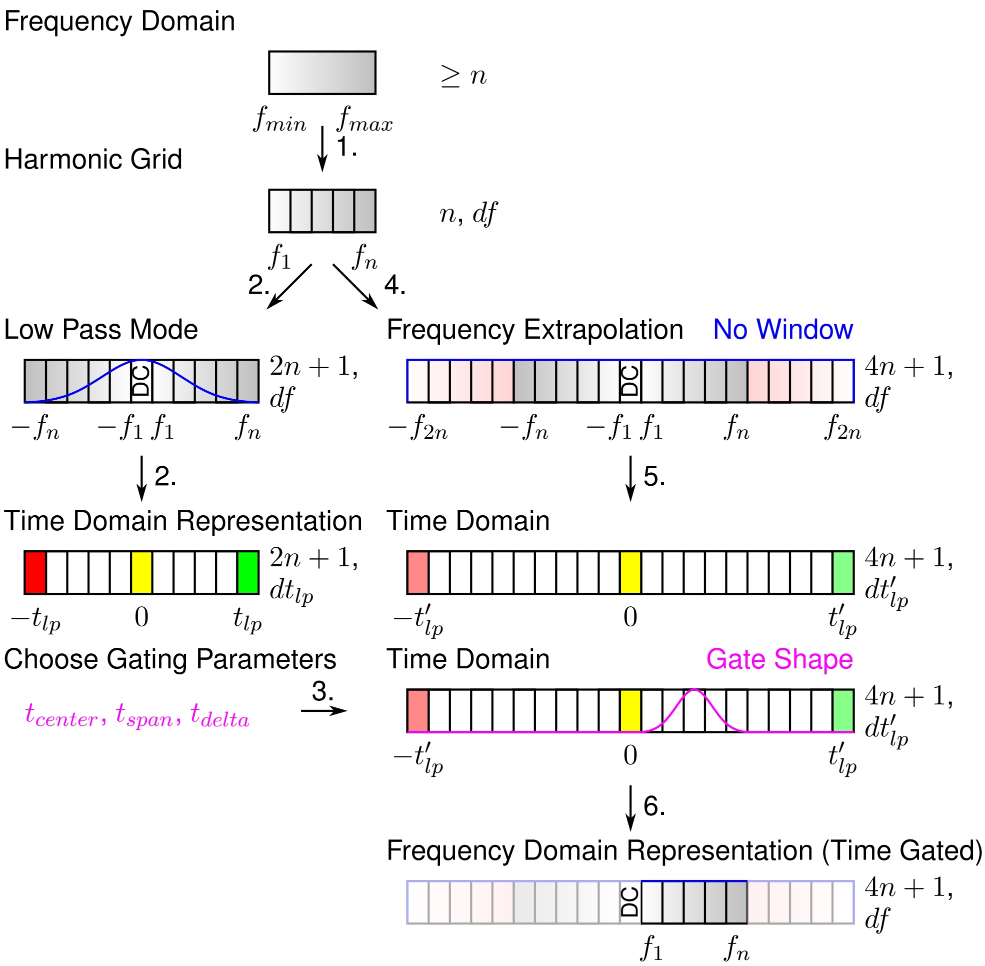

Figure 21: Illustration of time gating using the low pass mode. The steps 2 and 3 on the left side are used for time domain representation and to choose the gating parameters of the gate shape. The steps 4, 5 and 6 on the right side are used for the time gating process.

1. The original data in frequency domain is interpolated on a harmonic grid, see as well section B.1.
2. In the low pass mode, the data of the harmonic grid is mirrored (conjugate complex) to the negative frequencies and the DC point is added if required. The data on the harmonic grid is multiplied with the window function and then transformed to time domain using the inverse DFT (discrete fourier transform).
3. The gating parameters t_{center}, t_{span} and t_{delta} are chosen. See section B.2.3 for computing the gate shape.

4. The data on the harmonic grid is extrapolated at the higher negative and positive frequency ranges. The extrapolated points are a mirrored version of the harmonic grid and have large standard uncertainties of $\pm(1+|v_{re}|)/\sqrt{3}$ in real part and $\pm(1+|v_{im}|)/\sqrt{3}$ in imaginary part where $|v_p|$ is the absolute value of the part p. They are used to regularize the value and compute the uncertainty at the boundaries of the time gated data in frequency domain.

5. The extrapolated data in frequency domain is transformed to time domain without applying any window function.
   Note that the time resolution is $dt'_{lp} = \frac{2n+1}{4n+1} dt_{lp} \approx 0.5 dt_{lp}$.

6. The complex data in time domain from step 5 is multiplied with the gate shape and then transformed back to frequency domain. A subset of the frequencies is the result (time gated data in frequency domain).

B.2.3 Gate Shape

The gate shape with the parameters center time t_{center}, time span t_{span} and delta time t_{delta} is shown in figure 22.

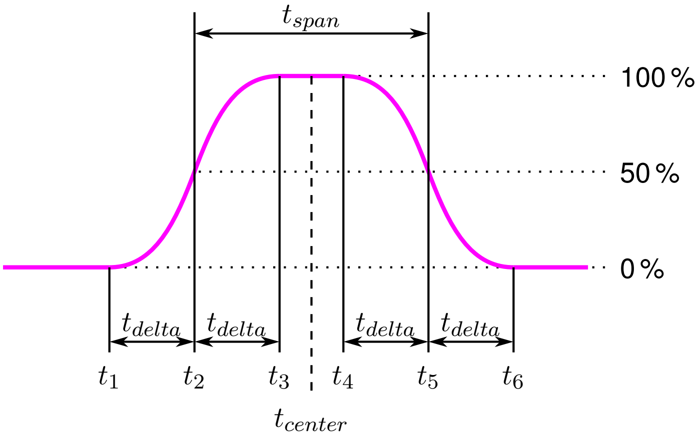

Figure 22: Gate shape

The gate shape $GS_{bandpass}$ is computed with the following function

$$GS_{bandpass} = \begin{cases} 0 & , t \leq t_1 \\ \mathrm{HannFilter}\left(t, t_1, t_3 + 2t_{delta}\right) & , t_1 < t < t_3 \\ 1 & , t_3 \leq t \leq t_4 \\ \mathrm{HannFilter}\left(t, t_4 - 2t_{delta}, t_6\right) & , t_4 < t < t_6 \\ 0 & , t_6 \leq t \end{cases} \tag{279}$$

and for a notch gate type

$$GS_{notch} = 1 - GS_{bandpass} \tag{280}$$

where the Hann filter is defined as

$$\mathrm{HannFilter}\left(t, t_{start}, t_{stop}\right) = 1 + \frac{\cos\left(2\pi \frac{t-(t_{start}+t_{stop})/2}{t_{stop}-t_{start}}\right)}{2} \tag{281}$$

between $t_{start} < t < t_{stop}$. Otherwise it returns 0.

C METAS UncLib

METAS UncLib [8], [9] is a generic measurement uncertainty calculator that supports the multivariate propagation of measurement uncertainty [7], taking correlations between quantities fully into account, see figure 23.

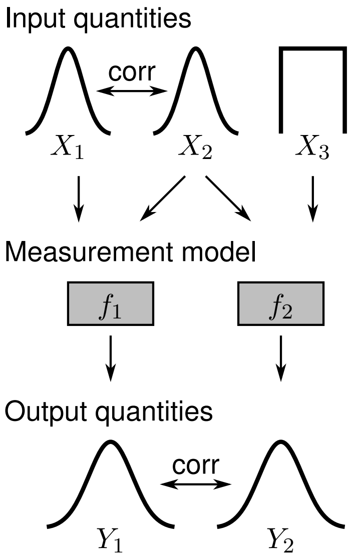

Figure 23: Illustration of multivariate uncertainty propagation: The uncertainties and correlations of basic input quantities $X_1,\ X_2,\ X_3$ are propagated through a measurement model resulting in uncertainties and correlations of multiple output quantities $Y_1,\ Y_2$, see [7].

The user only needs to specifies the input quantities $\mathbf{X}$ with uncertainties, or, more general with an input covariance matrix $\mathbf{V}_X$, and the measurement model $\mathbf{f}$. The actual propagation of uncertainty is done in the background in an automated way. METAS UncLib keeps automatically track of the derivatives with respect to the input quantities. Formally this means that METAS UncLib computes the output quantities $\mathbf{Y} = \mathbf{f}(\mathbf{X})$ and the Jacobi matrix $\mathbf{J}_{YX}$ of $\mathbf{f}$ that contains the drivatives of the components of $\mathbf{Y}$ with respect to the components of $\mathbf{X}$. On demand METAS UncLib can compute the output covariance matrix $\mathbf{V}_Y = \mathbf{J}_{YX}\mathbf{V}_X\mathbf{J}_{YX}'$. Further information about the technique behind METAS UncLib can be found in [9].

D Small Sample Statistics

The standard uncertainty associated with the final result is multiplied with the coverage factor to obtain an expanded uncertainty with a desired coverage probability (usually 95 %). The calculation of the coverage factor is not straightforward anymore, if the uncertainty contribution due to small sample statistics is significant. The GUM documents [6, 7] are not very consistent in how to treat such a situation, in particular for the multivariate case. We find neither the solution in GUM Supplement 2 with the multivariate t-distribution [7] nor the multivariate generalization of the Welch-Satterthwaite approach [26, 27] satisfactory. The following solution is a self-developed, pragmatic and safe approach to the problem. It will generally overestimate the uncertainty contribution due to small sample statistics.

A series of n measurements $(x_1,\ x_2,\ x_3, \ldots, x_n)$ of a vector quantity with length N leads to a sample covariance matrix of of the mean

$$\mathbf{S} = \frac{1}{n\,(n-1)} \left[(x_1 - \overline{x})\,(x_1 - \overline{x})' + \ldots + (x_n - \overline{x})\,(x_n - \overline{x})' \right]. \tag{282}$$

with $\overline{x}$ being the sample mean vector.

It is assumed that the n drawings are from a N-dimensional normal distribution. To obtain a $p{\cdot}100$ % confidence region the covariance matrix needs to be expanded with the following factor squared

$$k_{n,N,p} = \begin{cases} \mathrm{NormalDistCDF}^{-1}\left(\frac{p+1}{2}\right) & , N = 1 \wedge n = \infty \\ \mathrm{StudentTDistCDF}^{-1}\left(n-1, \frac{p+1}{2}\right) & , N = 1 \wedge n < \infty \\ \sqrt{\mathrm{Chi^2DistCDF}^{-1}\left(N, p\right)} & , N > 1 \wedge n = \infty \\ \sqrt{\frac{(n-1)N}{n-N}\,\mathrm{FDistCDF}^{-1}\left(n-N, N, p\right)} & , N > 1 \wedge n < \infty \end{cases} \tag{283}$$

Direct application of this factor can be done, if the covariance matrix is associated with the final result. If it is just an uncertainty contribution among others, which needs to be propagated to the end result, the following practical solution is applied. The sample covariance matrix of the mean $\mathbf{S}$ is extended using the factor

$$(f_{n,N,p})^2 = \left(\frac{k_{n,N,p}}{k_{\infty,N,p}}\right)^2, \tag{284}$$

leading to

$$\mathbf{T} = (f_{n,N,p})^2\,\mathbf{S}. \tag{285}$$

$\mathbf{T}$ is then used for subsequent uncertainty propagation to the end result. Finally, the covariance matrix associated with the end result is multiplied with $(k_{\infty,N,p})^2$.

Table 9 shows selected k- and f-factors for different numbers of measurement repetitions n and dimensions N.

Table 9: Coverage Factors

n	$k_{n,1,0.95}$	$k_{n,2,0.95}$	$k_{n,8,0.95}$	$f_{n,1,0.95}$	$f_{n,2,0.95}$	$f_{n,8,0.95}$
1						
2	12.7062			6.4829		
3	4.3027	28.2489		2.1953	11.5408	
4	3.1824	7.5498		1.6237	3.0844	
5	2.7764	5.0470		1.4166	2.0619	
6	2.5706	4.1666		1.3115	1.7022	
7	2.4469	3.7265		1.2484	1.5224	
8	2.3646	3.4642		1.2065	1.4153	
9	2.3060	3.2906	123.6466	1.1766	1.3444	31.3989
10	2.2622	3.1674	26.4075	1.1542	1.2940	6.7059
11	2.2281	3.0755	15.3582	1.1368	1.2565	3.9001
12	2.2010	3.0044	11.5284	1.1230	1.2274	2.9275
13	2.1788	2.9477	9.6183	1.1117	1.2042	2.4425
14	2.1604	2.9014	8.4781	1.1022	1.1853	2.1529
15	2.1448	2.8630	7.7209	1.0943	1.1696	1.9606
16	2.1314	2.8305	7.1813	1.0875	1.1564	1.8236
17	2.1199	2.8028	6.7773	1.0816	1.1450	1.7210
18	2.1098	2.7788	6.4633	1.0765	1.1352	1.6413
19	2.1009	2.7578	6.2122	1.0719	1.1267	1.5775
20	2.0930	2.7394	6.0068	1.0679	1.1191	1.5254
50	2.0096	2.5523	4.4984	1.0253	1.0427	1.1423
100	1.9842	2.4983	4.1914	1.0124	1.0206	1.0644
∞	1.9600	2.4477	3.9379	1.0000	1.0000	1.0000

The table helps to determine the improvements in accuracy that can be achieved by increasing the number of measurements. E.g. for $N = 2$ increasing the measurement repetitions from 4 to 5 leads to a reduction of the uncertainty contribution by approximately 2/3. The selected dimensions are based on the most often used cases:

$N = 1$: Scalar quantity

$N = 2$: Complex reflection factor of a 1-port DUT

$N = 8$: Complex S-matrix of a 2-port DUT

E Eigenvalue Problem

The most general problem is an over-determined non-linear eigenvalue problem

$$\mathbf{A_0 v} + \lambda \mathbf{A_1 v} + \lambda^2 \mathbf{A_2 v} + \ldots + \lambda^n \mathbf{A_n v} = 0. \tag{286}$$

The over-determined non-linear eigenvalue problem can be transformed to a balanced non-linear eigenvalue problem with order $m = 2n$ by squaring the over-determined non-linear problem

$$\underbrace{\mathbf{A_0^* A_0}}_{\mathbf{B_0}} \mathbf{v} + \lambda \underbrace{(\mathbf{A_0^* A_1} + \mathbf{A_1^* A_0})}_{\mathbf{B_1}} \mathbf{v} + \lambda^2 \underbrace{(\mathbf{A_0^* A_2} + \mathbf{A_1^* A_1} + \mathbf{A_2^* A_0})}_{\mathbf{B_2}} \mathbf{v} + \ldots + \lambda^m \underbrace{\mathbf{A_n^* A_n}}_{\mathbf{B_m}} \mathbf{v} = 0. \tag{287}$$

The operator * denotes the conjugate transpose.

This non-linear eigenvalue problem can then be rewritten as a linear eigenvalue problem by substitution

$$\underbrace{\begin{bmatrix} \mathbf{B_0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & & \mathbf{0} \\ \vdots & \vdots & & \ddots & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & -\mathbf{I} \end{bmatrix}}_{\mathbf{C_0}} \underbrace{\begin{bmatrix} \mathbf{v} \\ \lambda \mathbf{v} \\ \lambda^2 \mathbf{v} \\ \vdots \\ \lambda^m \mathbf{v} \end{bmatrix}}_{\mathbf{w}} + \lambda \underbrace{\begin{bmatrix} \mathbf{B_1} & \mathbf{B_2} & \mathbf{B_3} & \cdots & \mathbf{B_m} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & \mathbf{0} \end{bmatrix}}_{\mathbf{C_1}} \underbrace{\begin{bmatrix} \mathbf{v} \\ \lambda \mathbf{v} \\ \lambda^2 \mathbf{v} \\ \vdots \\ \lambda^m \mathbf{v} \end{bmatrix}}_{\mathbf{w}} = 0 \tag{288}$$

where $\mathbf{I}$ is the identity matrix.

This linear eigenvalue problem can finally be transformed to a standard eigenvalue problem

$$\underbrace{\mathbf{C_0}^{-1} \mathbf{C_1}}_{\mathbf{D}} \mathbf{w} = \underbrace{-\frac{1}{\lambda}}_{\lambda'} \mathbf{w}. \tag{289}$$

Note that $\mathbf{C_0}$ is only invertible if $\mathbf{B_0}$ is invertible. Otherwise the inverse of $\mathbf{C_1}$ is needed. This yields to the following standard eigenvalue problem

$$\underbrace{\mathbf{C_1}^{-1} \mathbf{C_0}}_{\mathbf{E}} \mathbf{w} = \underbrace{-\lambda}_{\lambda''} \mathbf{w}. \tag{290}$$

The eigenvalue computation with linear uncertainty propagation is described in [28] and it is fully implemented in METAS UncLib [8, 9].

F VNA Calibration Model Details

The following figures show the details of the VNA calibration models:

Figure 24 describes the one-port model.

Figure 25 describes an one-port calibration.

Figure 26 describes the difference between the generic and switched two-port model.

Figure 27 describes generic multi-port calibration model.

Figure 28 describes switched multi-port calibration model.

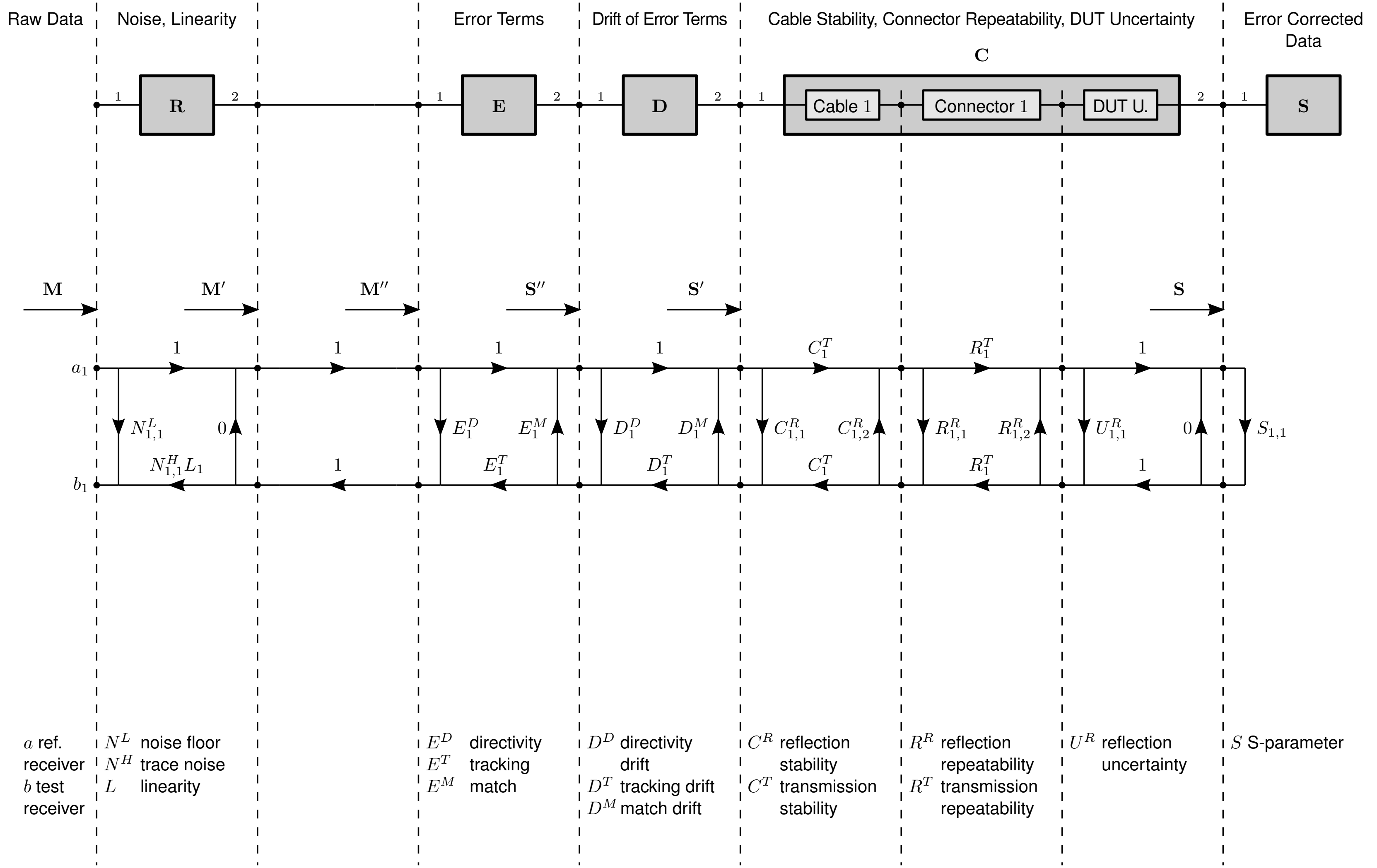

Figure 24: VNA 1-Port Calibration Model

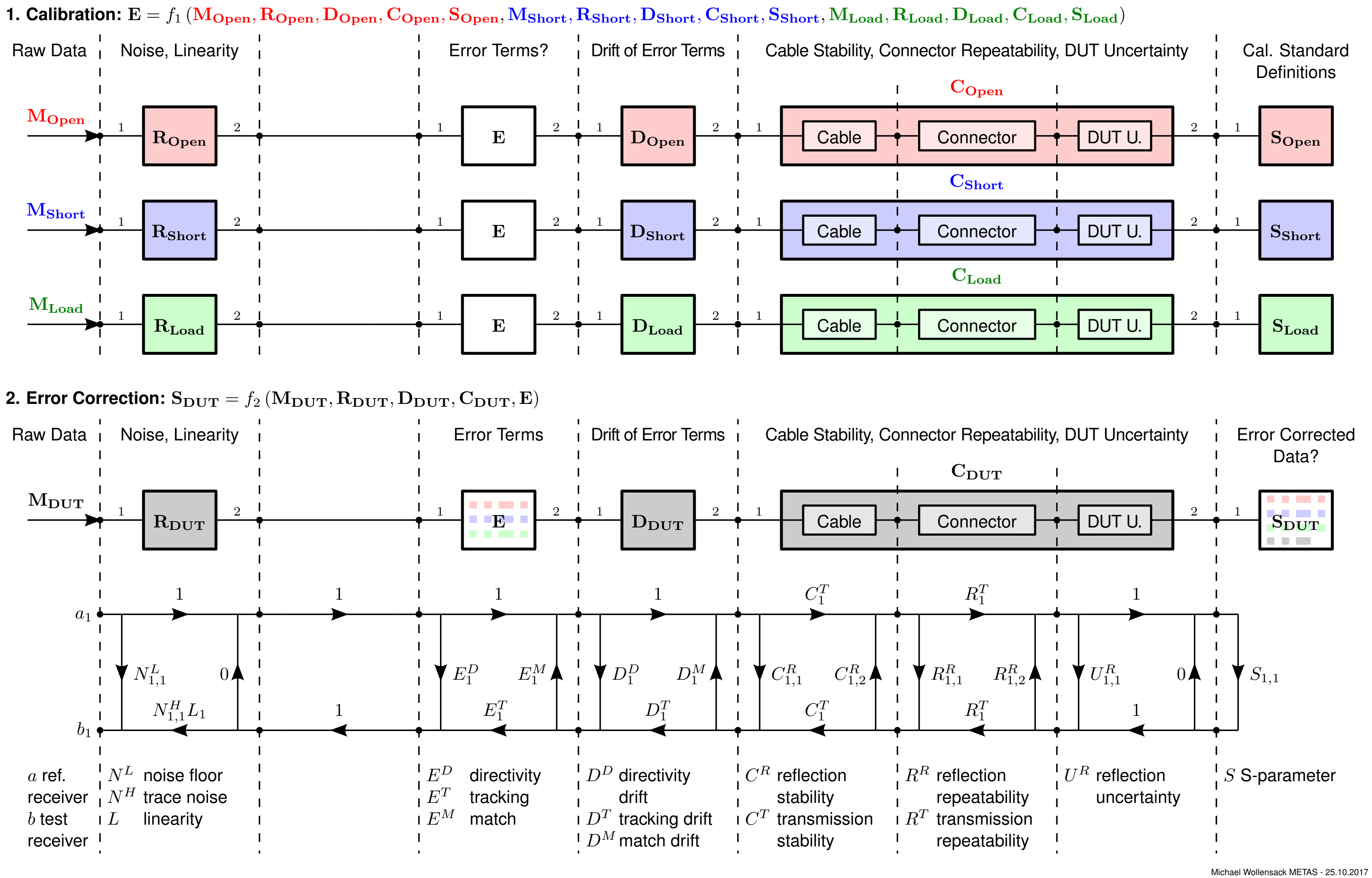

Figure 25: VNA 1-Port Calibration Model cont.

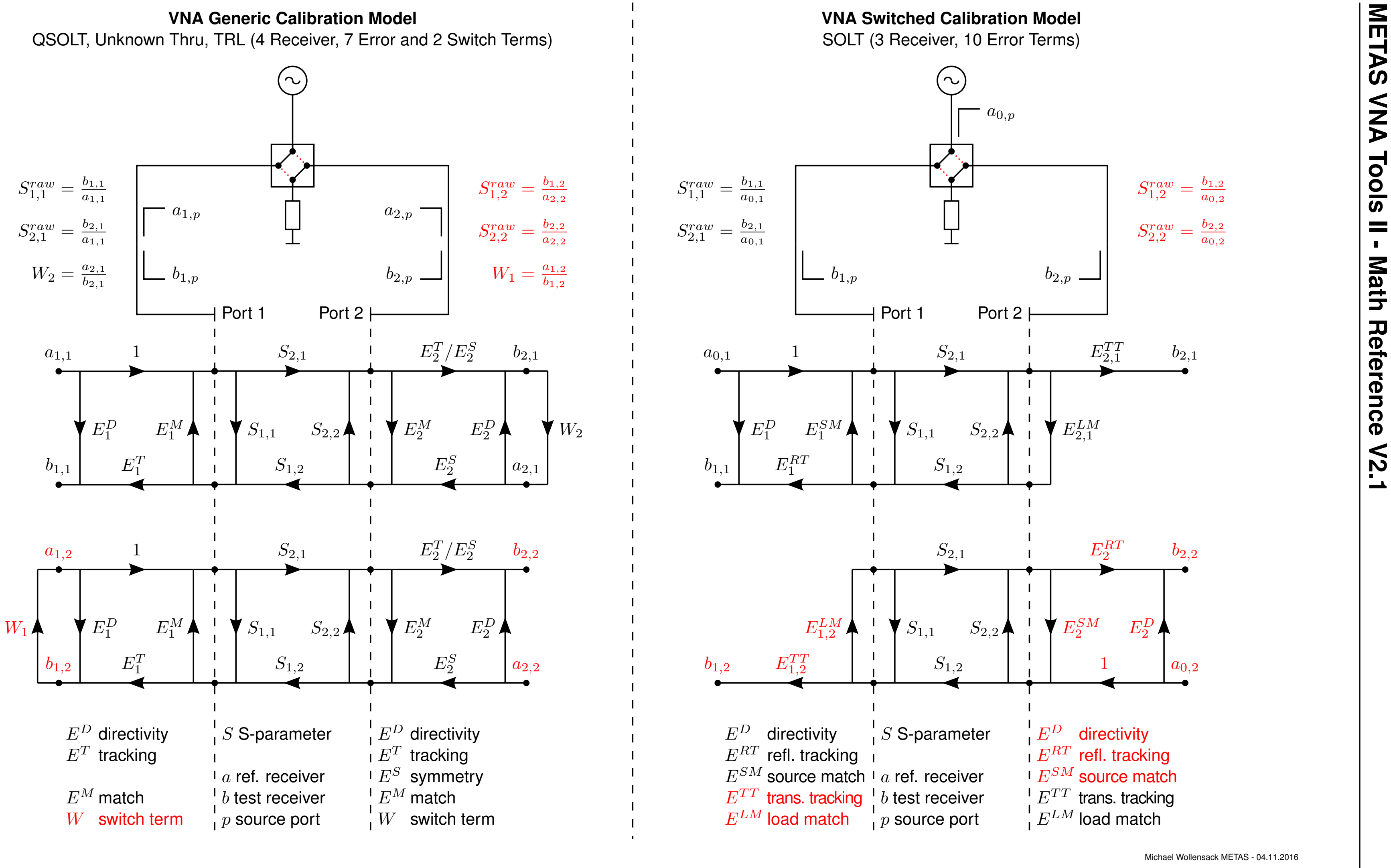

Figure 26: VNA 2-Port Calibration Model

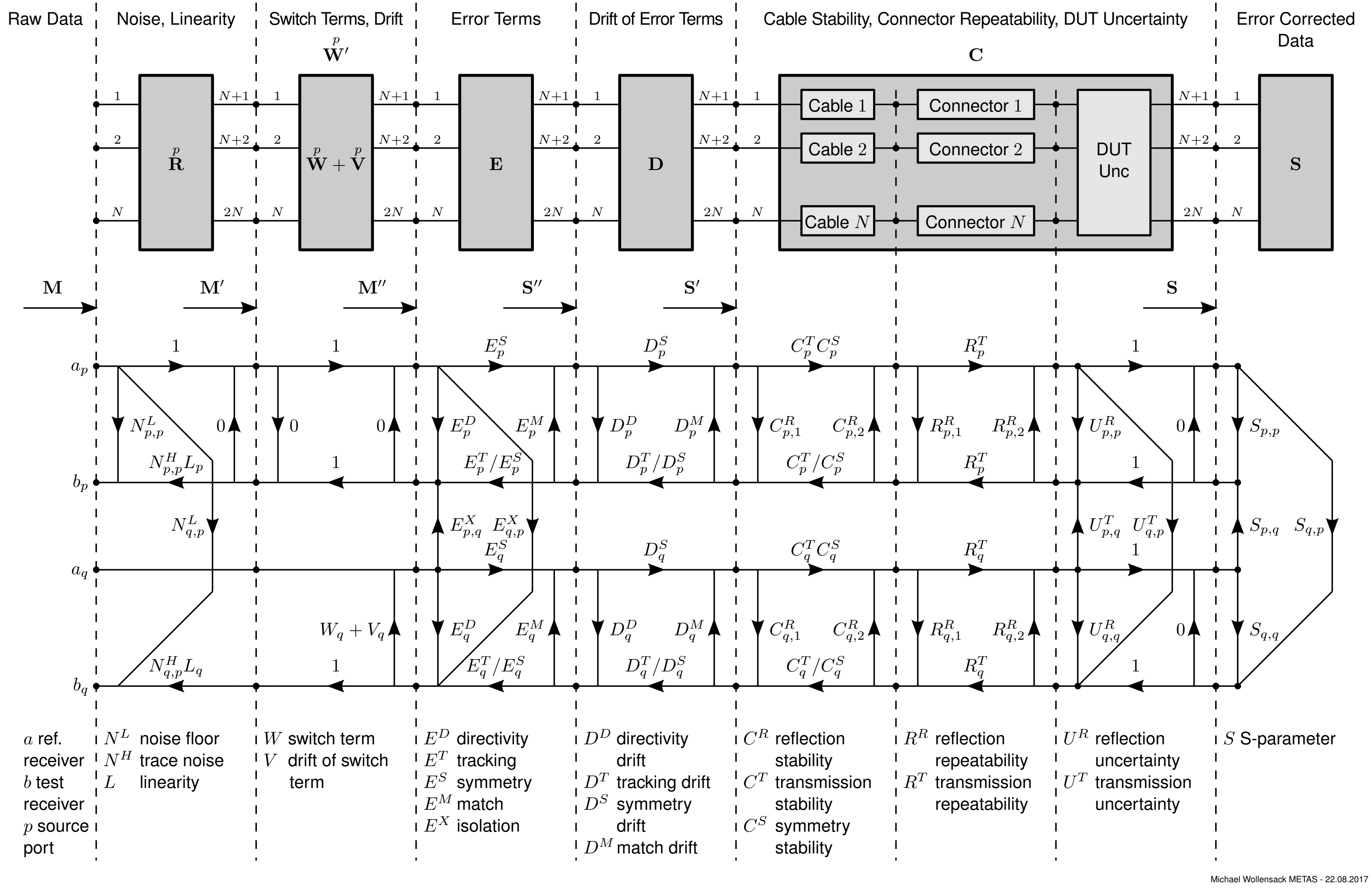

Figure 27: VNA Generic Calibration Model

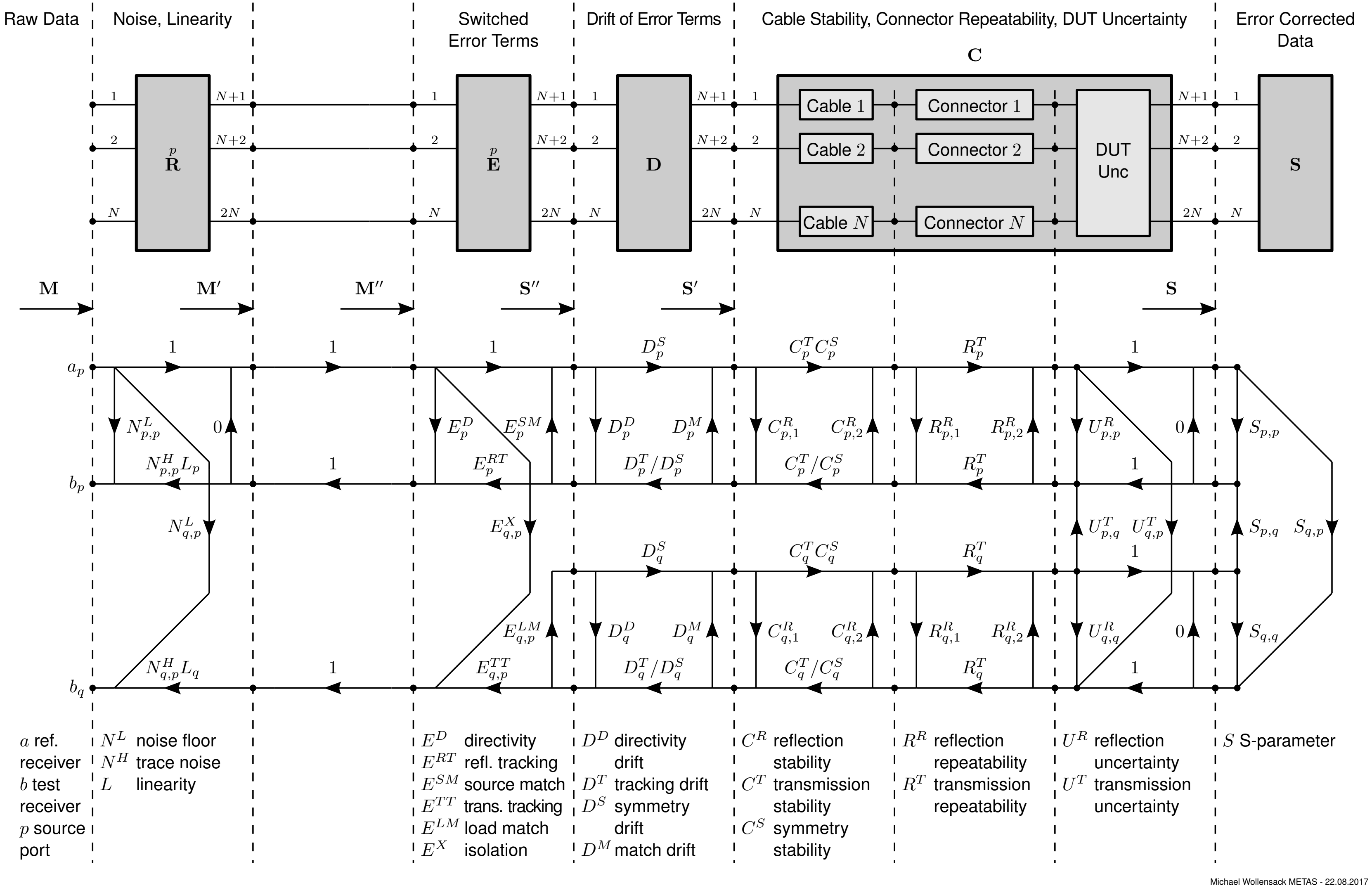

Figure 28: VNA Switched Calibration Model

References

[1] K. Wong, “Traceability of vector network analyzer measurements”, in *72th ARFTG Conf. Dig.*, pp. 157-167, December 2008

[2] *Guidelines on the Evaluation of Vector Network Analysers (VNA)*, European Association of National Metrology Institutes (EURAMET), cg-12, Version 2.0, March 2011.

[3] J. Hoffmann, P. Leuchtmann, J. Ruefenacht, and K. Wong, “S-parameters of Slotted and Slotless Coaxial Connectors”, in *74th ARFTG Conf. Dig.*, pp. 1-5, December 2009

[4] D. F. Williams, J. C. M. Wang, and U. Arz, “An optimal vector-network-analyzer calibration algorithm”, *IEEE Trans. Microwave Theory & Tech.*, vol. 51, no. 12, pp. 987-993, December 2003.

[5] D. K. Rytting, “Network Analyzer Accuracy Overview ”, in *58th ARFTG Conf. Dig.*, pp. 1-13, November 2001

[6] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, *Evaluation of measurement data - Guide to the expression of uncertainty in measurement*, International Organization for Standardization (ISO), Online: http://www.bipm.org/en/publications/guides/gum.html, September 2008

[7] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, *Evaluation of measurement data - Supplement 2 to the ”Guide to the expression of uncertainty in measurement” - Extension to any number of output quantities*, International Organization for Standardization (ISO), Online: http://www.bipm.org/en/publications/guides/gum.html, October 2011

[8] M. Zeier, J. Hoffmann, and M. Wollensack, “Metas.UncLib - a measurement uncertainty calculator for advanced problems”, *Metrologia*, vol. 49, no. 6, pp. 809-815, Online: http://stacks.iop.org/0026-1394/49/i=6/a=809, November 2012.

[9] M. Wollensack, *METAS UncLib*, Online: http://www.metas.ch/unclib, 2009.

[10] M. Wollensack, J. Hoffmann, J. Ruefenacht, and M. Zeier, “VNA Tools II: S-parameter uncertainty calculation”, in *79th ARFTG Conf. Dig.*, pp. 1-5, June 2012

[11] A. Ferrero, and U. Pisani, “QSOLT: a new fast calibration algorithm for two port S-parameter measurements”, in *38th ARFTG Conf. Dig.*, pp. 15-24, December 1991.

[12] A. Ferrero, and U. Pisani, “Two-port network analyzer calibration using an unknown thru”, *IEEE Microwave & Guided Wave Letters*, vol. 2, no. 12, pp. 505-507, December 1992.

[13] G. F. Engen, and C. A. Hoer, “Thru-reflect-line: An improved technique for calibrating the dual six-port automatic network analyzer”, *IEEE Trans. Microwave Theory & Tech.*, vol. 27, no. 12, pp. 987-993, December 1979.

[14] F. Purroy and L. Pradell, “New theoretical analysis for the LRRM calibration technique for vector network analyzers”, *IEEE Trans. Microwave Theory & Tech.*, vol. 50, no. 5, pp. 1307-1314, October 2001.

[15] J. R. Juroshek, “A direct calibration method for measuring equivalent source mismatch”, *Microwave Journal*, vol. 40, no. 10, pp. 106-118, October 1997

[16] A. Ferrero, V. Teppati, M. Garelli and A. Neri, “A Novel Calibration Algorithm for a Special Class of Multiport Vector Network Analyzers”, *IEEE Trans. Microwave Theory & Tech.*, vol. 56, no. 3, pp. 693-699, March 2008.

[17] M. Wollensack, J. Hoffmann, J. Ruefenacht, and M. Zeier, “VNA Tools II: Calibrations Involving Eigenvalue Problems”, in *89th ARFTG Conf. Dig.*, pp. 1-4, June 2017

[18] R. B. Marks, “Formulations of the basic vector network analyzer error model including switch-terms”, in *50th ARFTG Conf. Dig.*, pp. 115-126, December 1997

[19] J. Hoffmann, P. Leuchtmann, J. Ruefenacht, and R. Vahldieck, “A stable bayesian vector network analyzer calibration algorithm”, *IEEE Trans. Microwave Theory & Tech.*, vol. 57, no. 4, pp. 869-880, April 2009.

[20] Agilent Technologies, *Specifying Calibration Standards and Kits for Agilent Vector Network Analyzers*, Application Note 1287-11, Online: http://cp.literature.agilent.com/litweb/pdf/5989-4840EN.pdf, March 2011.

[21] W. Heinrich, “Quasi-TEM Description of MMIC Coplanar Lines Including Conductor-Loss Effects”, *IEEE Trans. Microwave Theory & Tech.*, vol. 41, no. 1, pp. 45-52, January 1993.

[22] A. Ferrero, U. Pisani, and K. J. Kerwin, “A new implementation of a multiport automatic network analyzer”, *IEEE Trans. Microwave Theory & Tech.*, vol. 40, no. 11, pp. 2078-2085, November 1992.

[23] R. B. Marks, and D. F. Williams, “A general waveguide circuit theory”, *Journal of Research of the National Institute of Standards and Technology*, vol. 97, no. 5, pp. 533-562, October 1992.

[24] W. C. Daywitt, “First-order symmetric modes for a slightly lossy coaxial transmission line”, *IEEE Trans. Microwave Theory & Tech.*, vol. 38, no. 11, pp. 1644-1650, November 1990.

[25] Keysight Technologies, *Time Domain Analysis using a Network Analyzer*, Application Note 1287-12, Online: http://literature.cdn.keysight.com/litweb/pdf/5989-5723EN.pdf, August 2014.

[26] R. Willink, and B. D. Hall, “A classical method for uncertainty analysis with multidimensional data”, *Metrologia*, vol. 39, no. 4, pp. 361-369, 2002.

[27] R. Willink, and B. D. Hall, “An extension to GUM methodology: degrees-of-freedom calculations for correlated multidimensional estimates”, Online: http://arxiv.org/abs/1311.0343, 2013.

[28] J. R. Magnus, “On Differentiating Eigenvalues and Eigenvectors”, *Econometric Theory*, vol. 1, no. 2, pp. 179-191, Online: http://janmagnus.nl/papers/JRM011.pdf, August 1985