# METAS UncLib Python - User Reference V2.6.0

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## Contents

# 1 Introduction

This document is a quick reference sheet. For practical demonstrations and more details refer to the tutorial and the examples that are provided with the installation of the software.

The METAS UncLib Python library is an extension to Python, which supports creation of uncertainty objects and subsequent calculation with them as well as storage of the results. It's able to handle complex-valued and multivariate quantities. It has been developed with Python V3.6 using the numpy (1.16.1) and the pythonnet (2.3.0) packages. It requires the C# library METAS UncLib in the background. There are three modules for uncertainty propagation: `LinProp`, `DistProp` and `MCProp`.

**LinProp** supports linear uncertainty propagation $\mathbf{V}_{out} = \mathbf{J}\mathbf{V}_{in}\mathbf{J}'$.

**DistProp** supports higher order uncertainty propagation, i.e. higher order terms of the Taylor expansion of the measurement equation are taken into account.[1]

**MCProp** supports Monte Carlo propagation.[1]

# 2 Global uncertainty settings

`from metas_unclib import *` Import METAS UncLib.

`use_linprop()` Use the linear uncertainty propagation.

`use_distprop(maxlevel=1)` Use the higher order uncertainty propagation.
The argument `maxlevel` specifies the higher order uncertainty propagation maximum level. Default value: 1 (1 corresponds to `LinProp`)

`use_mcprop(n=100000)` Use the Monte Carlo uncertainty propagation.
The argument `n` specifies the Monte Carlo uncertainty propagation sample size. Default value: 100000

[1] preliminary implementation

## 3 Create an uncertainty object

Square brackets indicate vector or matrix.

`x = ufloat(value)` Creates a new uncertain number without uncertainties.

`x = ufloat(value, stdunc, idof=0.0, desc=None)` Creates a new real uncertain number with value, standard uncertainty, inverse degrees of freedom (optional), and a description (optional).

`x = ucomplex(value, [covariance], desc=None)` Creates a new complex uncertain number. Covariance size: $2x2$

`x = ufloatarray([value], [covariance], desc=None)` Creates a new real uncertain array. Covariance size: $NxN$

`x = ucomplexarray([value], [covariance], desc=None)` Creates a new complex uncertain array.

`x = ufloatfromsamples([samples], desc=None, p=0.95)` Creates a new real uncertain number from samples with a description (optional) and a probability (optional).

`x = ucomplexfromsamples([samples], desc=None, p=0.95)` Creates a new complex uncertain number from samples with a description (optional) and a probability (optional). The complex uncertain number contains the correlation between real and imaginary parts.

`x = ufloatarrayfromsamples([samples], desc=None, p=0.95)` Creates a new real uncertain array from samples with a description (optional) and a probability (optional). The real uncertain array contains the correlation between the different entries.

`x = ucomplexarrayfromsamples([samples], desc=None, p=0.95)` Creates a new complex uncertain array from samples with a description (optional) and a probability (optional). The complex uncertain array contains the correlation between real and the imaginary parts and the different entries.

`x = ufloatsystem(value, [sys_inputs],[sys_sensitivities])` Creates a new real uncertain number by setting sensitivities with respect to uncertain inputs.[2]

[2] `LinProp` uncertainty objects only

# 4 Calculations with uncertainty objects

## 4.1 Math functions

- `x + y`
- `x - y`
- `umath.sqrt(x)`
- `umath.exp(x)`
- `umath.log(x)`
- `umath.log10(x)`
- `umath.ellipk(x)`
- `umath.ellipe(x)`
- `x * y`
- `x / y`
- `umath.sin(x)`
- `umath.cos(x)`
- `umath.tan(x)`
- `umath.asin(x)`
- `umath.acos(x)`
- `umath.atan(x)`
- `x ** y` [3]
- `-x`
- `umath.sinh(x)`
- `umath.cosh(x)`
- `umath.tanh(x)`
- `umath.asinh(x)`
- `umath.acosh(x)`
- `umath.atanh(x)`
- `umath.pow(x, y)`
- `umath.real(x)`
- `umath.imag(x)`
- `umath.abs(x)`
- `umath.angle(x)`
- `umath.conj(x)`

## 4.2 Linear algebra

`ulinalg.dot(M1, M2)` Matrix multiplication of matrix $\mathbf{M_1}$ and $\mathbf{M_2}$

`ulinalg.det(M)` Determinate of matrix $\mathbf{M}$

`ulinalg.inv(M)` Matrix inverse of $\mathbf{M}$

`ulinalg.solve(A, Y)` Solve linear equation system: $\mathbf{Ax} = \mathbf{y}$

`ulinalg.lstsqrsolve(A, Y)` Least square solve over determined equation system

`ulinalg.weightedlstsqrsolve(A, Y, W)` Weighted least square solve over determined equation system

`V, D = ulinalg.eig(A0)` Eigenvalue problem[2]: $\mathbf{A_0 V} = \mathbf{VD}$

`V, D = ulinalg.eig(A0, A1, A2, ..., An-1)` Non-linear eigenvalue problem[2]: $\mathbf{A_0 V} + \mathbf{A_1 V D} + \mathbf{A_2 V D}^2 + \ldots + \mathbf{A_{(n-1)} V D}^{(n-1)} = 0$

[2] `LinProp` uncertainty objects only
[3] `**` is the power operator

## 4.3 Numerical routines

`unumlib.polyfit(x, y, n)` Fit polynom to data

`unumlib.polyval(p, x)` Evaluate polynom

`unumlib.interpolation(x, y, n, xx)` Interpolation

`unumlib.interpolation2(x, y, n, xx)` Interpolation with linear uncertainty propagation

`unumlib.splineinterpolation(x, y, xx, boundaries)` Spline interpolation

`unumlib.splineinterpolation2(x, y, xx, boundaries)` Spline interpolation with linear uncertainty propagation

`unumlib.integrate(x, y, n)` Integrate

`unumlib.splineintegrate(x, y, boundaries)` Spline integrate

`unumlib.fft(v)` Fast Fourier transformation

`unumlib.ifft(v)` Inverse Fast Fourier transformation

`unumlib.dft(v)` Discrete Fourier transformation[2]

`unumlib.idft(v)` Inverse discrete Fourier transformation[2]

`unumlib.numerical_step(@f, x, dx)` Numerical step[2]

`unumlib.optimizer(@f, xStart, p)` Optimizer[2]

[2] `LinProp` uncertainty objects only

# 5 Get properties of an uncertainty object

`get_value(y)` Returns the expected value.

`get_fcn_value(y)` Returns the function value.

`get_stdunc(y)` Computes the standard uncertainty.

`get_coverage_interval(y, p)` Computes the coverage interval.

`get_moment(y, n)` Computes the n-th central moment.

`get_correlation([y1 y2 ...])` Computes the correlation matrix.

`get_covariance([y1 y2 ...])` Computes the covariance matrix.

`get_idof(y)` Computes the inverse degrees of freedom.[2]

`1.0 / get_idof(y)` Computes the degrees of freedom.[2]

`get_jacobi(y)` Returns the sensitivities to the virtual base inputs (with value 0 and uncertainty 1).[2]

`get_jacobi2(y, x)` Computes the sensitivities of $\mathbf{y}$ to the intermediate results $\mathbf{x}$.[2]

`get_unc_component(y, x)` Computes the uncertainty components of $\mathbf{y}$ with respect to $\mathbf{x}$.[2]

`unc_budget(y)` Shows the uncertainty budget.[2]

# 6 Storage functions

## 6.1 Store a computed uncertainty object

`ustorage.save_binary_file(y, filepath)` Binary serializes uncertainty object $\mathbf{y}$ to file.

`ustorage.save_xml_file(y, filepath)` XML serializes uncertainty object $\mathbf{y}$ to file.

`ustorage.to_xml_string(y)` XML serializes uncertainty object $\mathbf{y}$ to string.

## 6.2 Reload a stored uncertainty object

`ustorage.load_binary_file(filepath)` Reloads uncertainty object from binary file.

`ustorage.load_xml_file(filepath)` Reloads uncertainty object from XML file.

`ustorage.from_xml_string(s)` Reloads uncertainty object from XML string.

[2] `LinProp` uncertainty objects only

# A Physical constants

`uconst` is equal to the newest physical constants `uconst2018`, see subsection A.3.

## A.1 CODATA 2014

The following list contains the exact physical constants:

`uconst2014.deltavCs` Hyperfine transition frequency of Cs-133 in Hz

`uconst2014.c0` Speed of light in vacuum in m/s

`uconst2014.mu0` Vacuum magnetic permeability in Vs/Am

`uconst2014.ep0` Vacuum electric permittivity in As/Vm

`uconst2014.Kcd` Luminous efficacy in lm/W

`uconst2014.Mu` Molar mass constant in kg/mol

The following list contains the physical constants with uncertainties:

`uconst2014.G` Newtonian constant of gravitation[3] in $m^3/(kg*s^2)$

`uconst2014.alpha` Fine-structure constant[3]

`uconst2014.Ryd` Rydberg constant[3] in 1/m

`uconst2014.mpsme` Proton-electron mass ratio[3]

`uconst2014.Na` Avogadro constant[3] in 1/mol

`uconst2014.Kj` Josephson constant[3] in Hz/V

`uconst2014.k` Boltzmann constant[3] in J/K

`uconst2014.Rk` von Klitzing constant in Ohm

`uconst2014.e` Elementary charge in C

`uconst2014.h` Planck constant in Js

`uconst2014.me` Electron mass in kg

`uconst2014.mp` Proton mass in kg

`uconst2014.u` Atomic mass constant in kg

`uconst2014.F` Faraday constant in C/mol

`uconst2014.R` Molar gas constant in J/(mol*K)

`uconst2014.eV` Electron volt in J

[3]The correlation matrix of this physical constants is used in METAS UncLib to generate uncertainty objects which are correlated. The other physical constants are computed out of this set and the exact physical constants, e.g.: `Rk = mu0*c0/(2*alpha)` and `e = 2/(Kj*Rk)`.

### A.2 CODATA 2014 for conventional electrical units 90

The following list contains the exact physical constants:

`uconst2014_90.deltavCs` Hyperfine transition frequency of Cs-133 in Hz

`uconst2014_90.c0` Speed of light in vacuum in m/s

`uconst2014_90.mu0` Vacuum magnetic permeability in Vs/Am

`uconst2014_90.ep0` Vacuum electric permittivity in As/Vm

`uconst2014_90.Kcd` Luminous efficacy in lm/W

`uconst2014_90.Mu` Molar mass constant in kg/mol

`uconst2014_90.Kj` Conventional value of Josephson constant in Hz/V

`uconst2014_90.Rk` Conventional value of von Klitzing constant in Ohm

`uconst2014_90.e` Elementary charge in C

`uconst2014_90.h` Planck constant in Js

The following list contains the physical constants with uncertainties:

`uconst2014_90.Na` Avogadro constant in 1/mol

`uconst2014_90.F` Faraday constant in C/mol

`uconst2014_90.k` Boltzmann constant in J/K

## A.3 CODATA 2018

The following list contains the exact physical constants:

`uconst2018.deltavCs` Hyperfine transition frequency of Cs-133 in Hz

`uconst2018.c0` Speed of light in vacuum in m/s

`uconst2018.h` Planck constant in Js

`uconst2018.e` Elementary charge in C

`uconst2018.k` Boltzmann constant in J/K

`uconst2018.Na` Avogadro constant in 1/mol

`uconst2018.Kcd` Luminous efficacy in lm/W

`uconst2018.Kj` Josephson constant in Hz/V

`uconst2018.Rk` von Klitzing constant in Ohm

`uconst2018.F` Faraday constant in C/mol

`uconst2018.R` Molar gas constant in J/(mol*K)

`uconst2018.eV` Electron volt in J

The following list contains the physical constants with uncertainties:

`uconst2018.G` Newtonian constant of gravitation[4] in $m^3/(kg*s^2)$

`uconst2018.alpha` Fine-structure constant[4]

`uconst2018.mu0` Vacuum magnetic permeability in Vs/Am

`uconst2018.ep0` Vacuum electric permittivity in As/Vm

`uconst2018.Ryd` Rydberg constant[4] in 1/m

`uconst2018.me` Electron mass in kg

`uconst2018.are` Electron relative atomic mass[4]

`uconst2018.arp` Proton relative atomic mass[4]

`uconst2018.mpsme` Proton-electron mass ratio

`uconst2018.mp` Proton mass in kg

`uconst2018.u` Atomic mass constant in kg

`uconst2018.Mu` Molar mass constant in kg/mol

[4] The correlation matrix of this physical constants is used in METAS UncLib to generate uncertainty objects which are correlated. The other physical constants are computed out of this set and the exact physical constants, e.g.: `mu0 = 2*h/(e*e*c0)*alpha` and `ep0 = 1.0/(c0*c0*mu0)`.